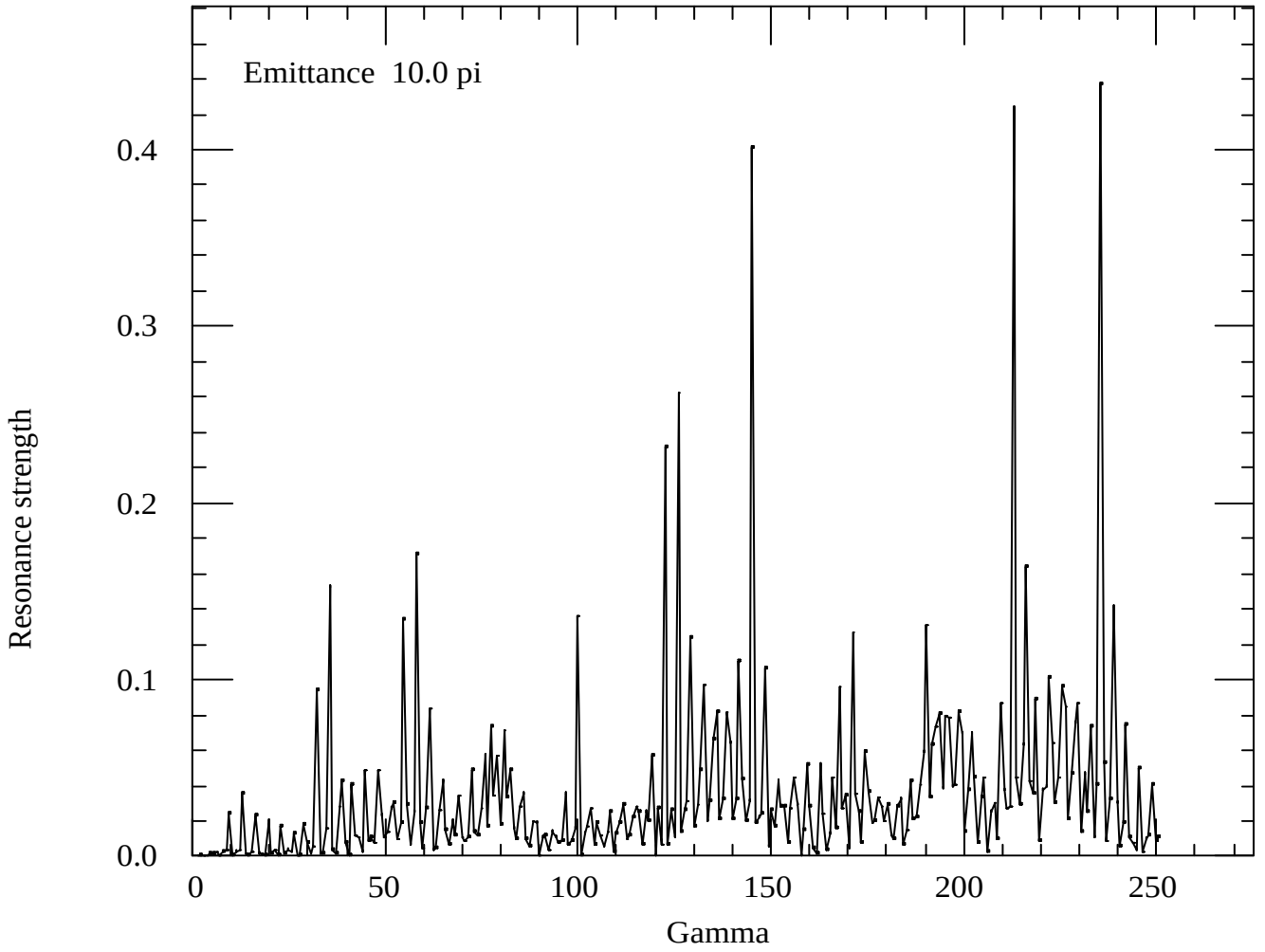
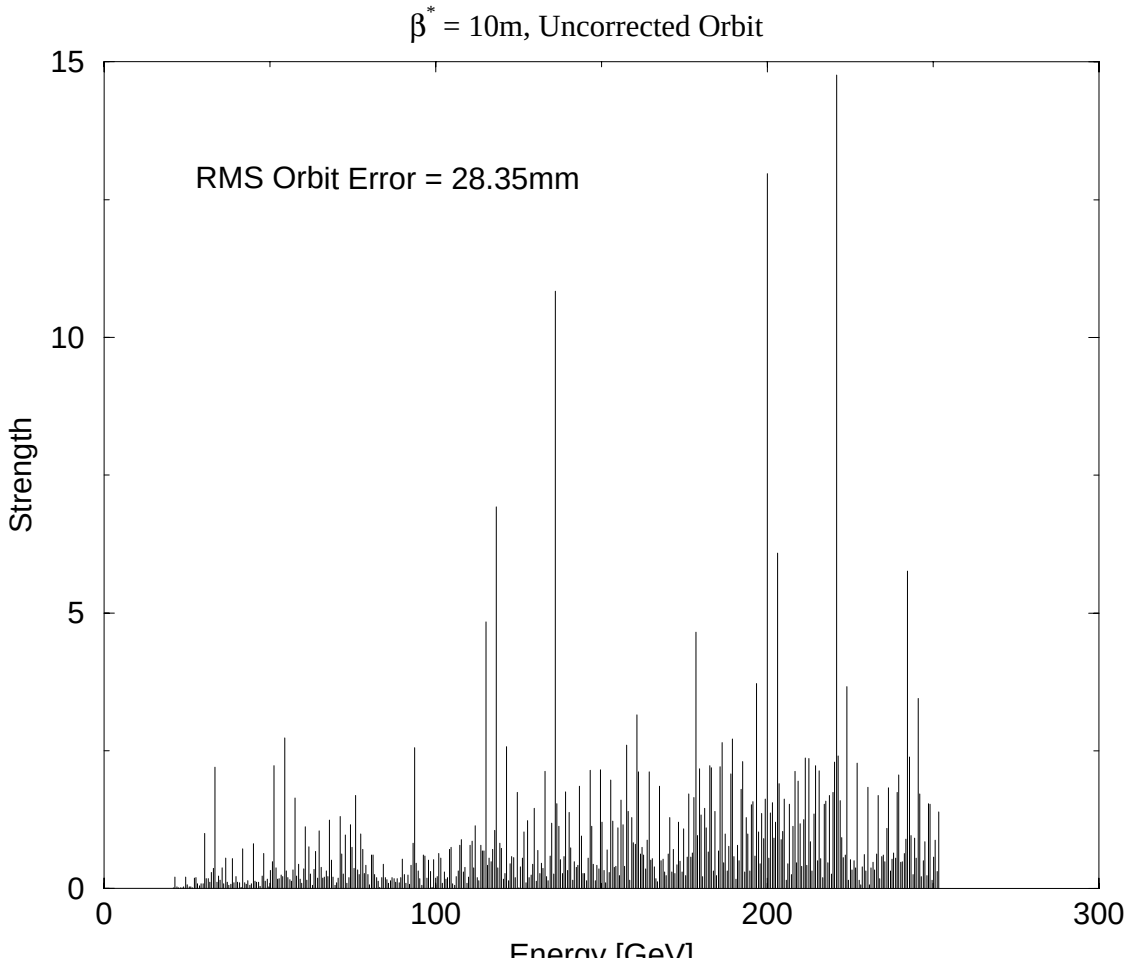
Intrinsic resonance strengths, protons in RHIC

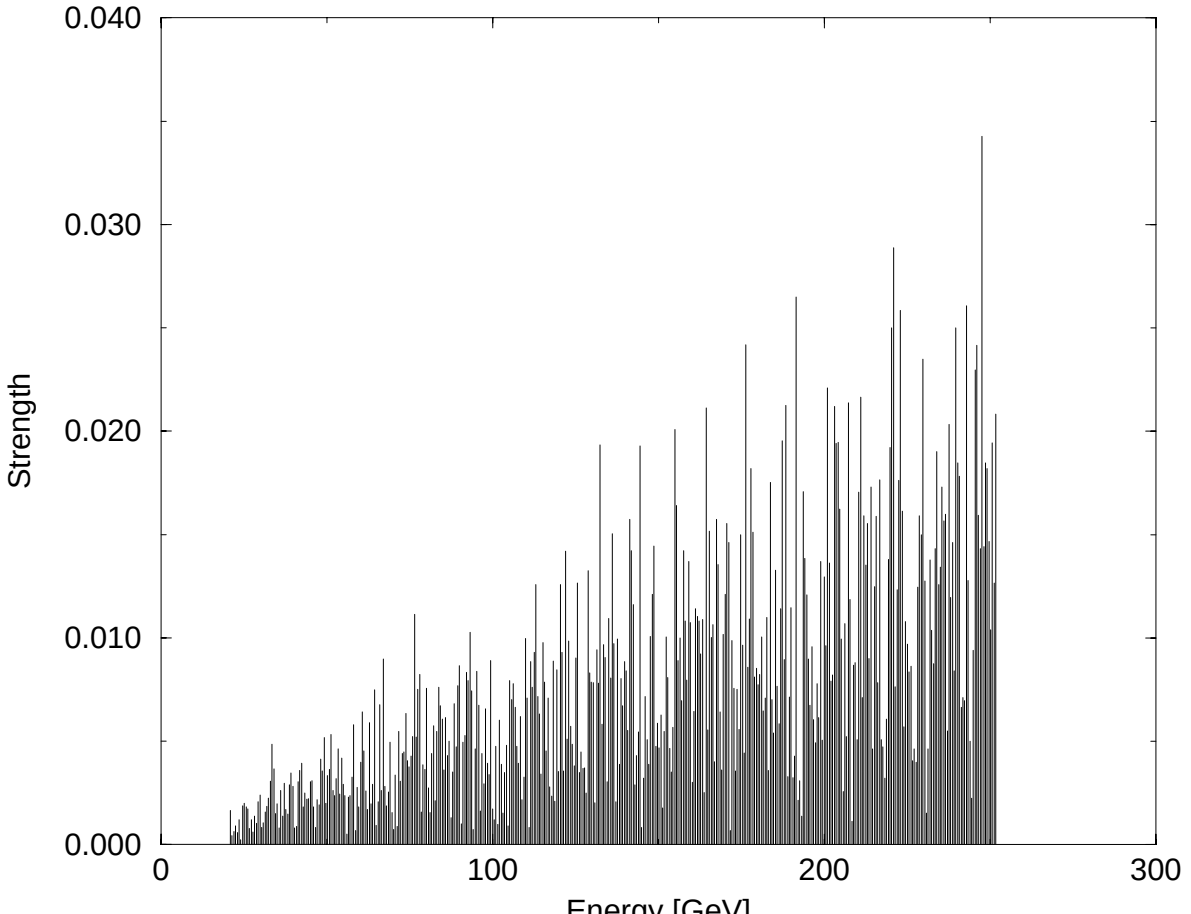


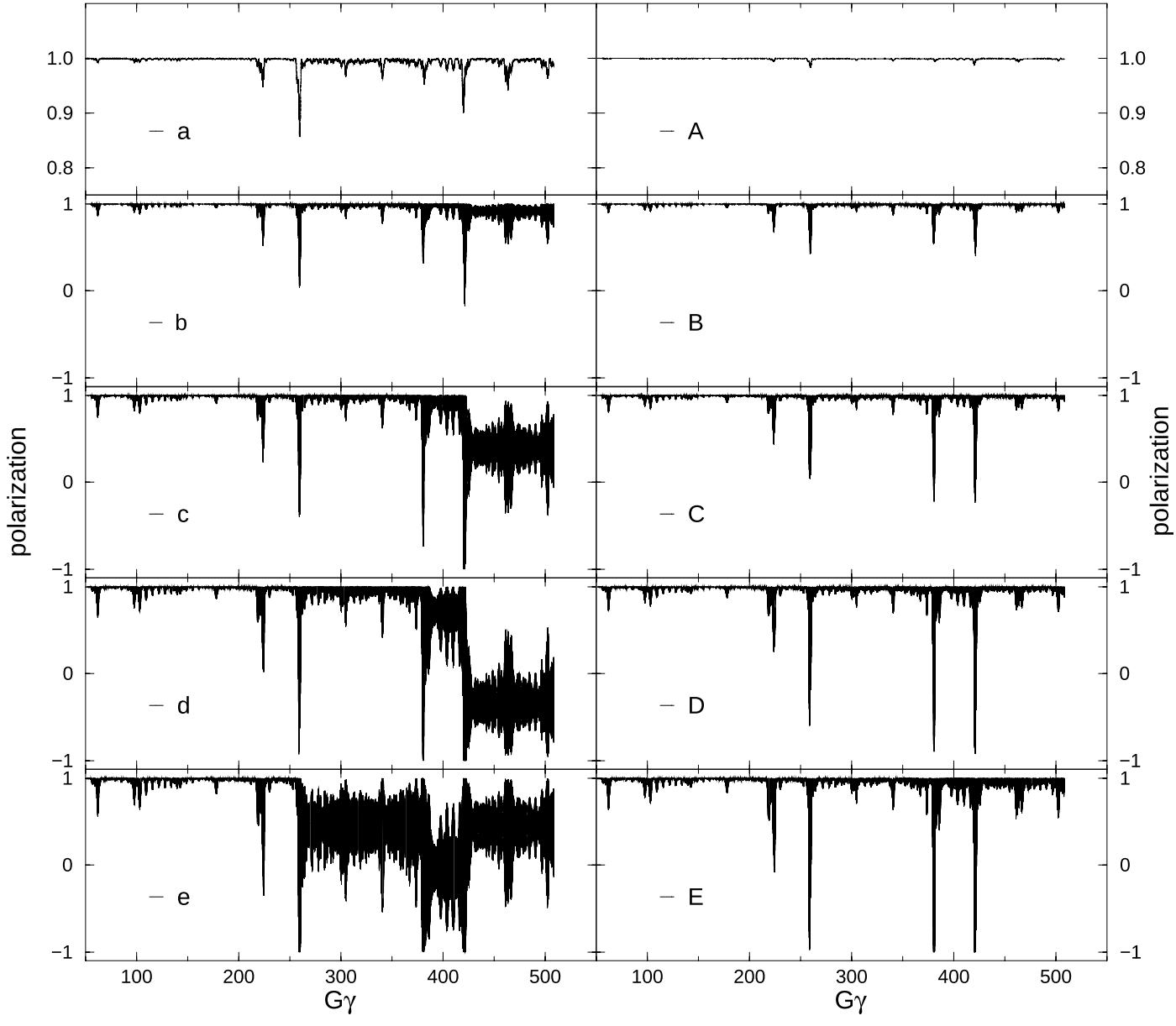
Imperfection Depolarizing Resonances

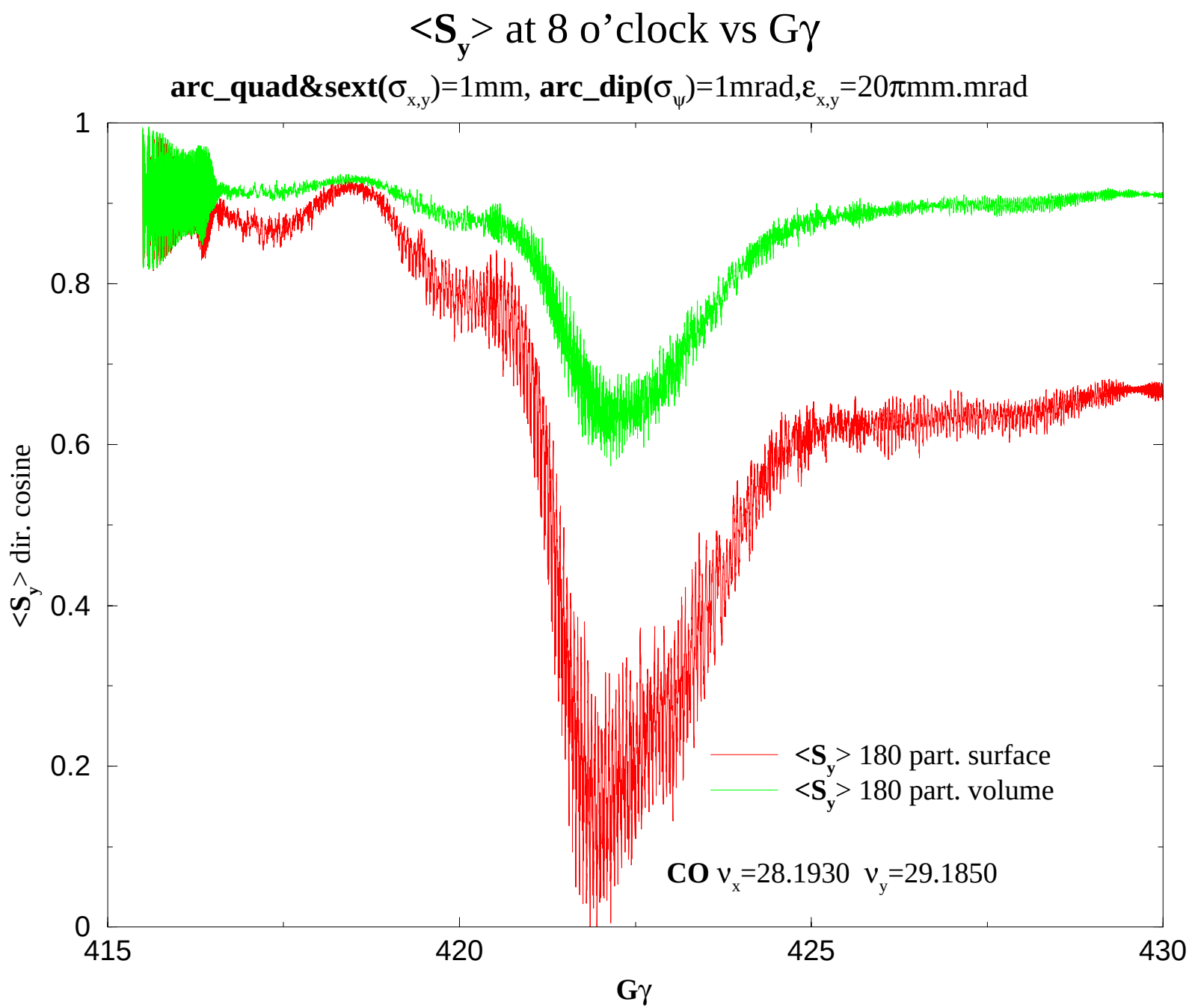


Imperfection Depolarizing Resonances

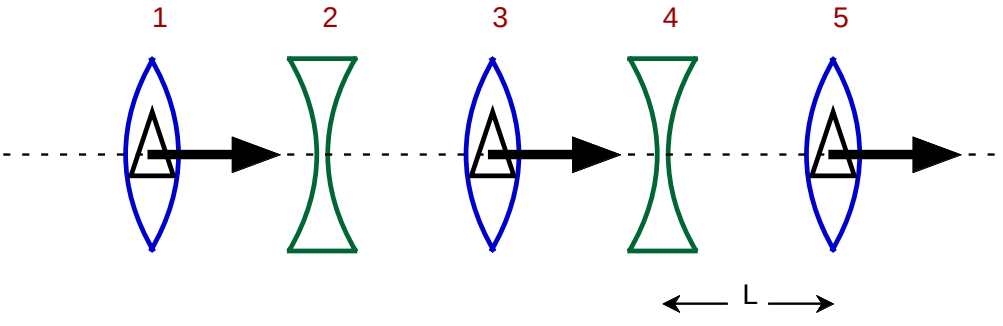
$\beta^* = 10$m, Orbit Error = 0.155mm



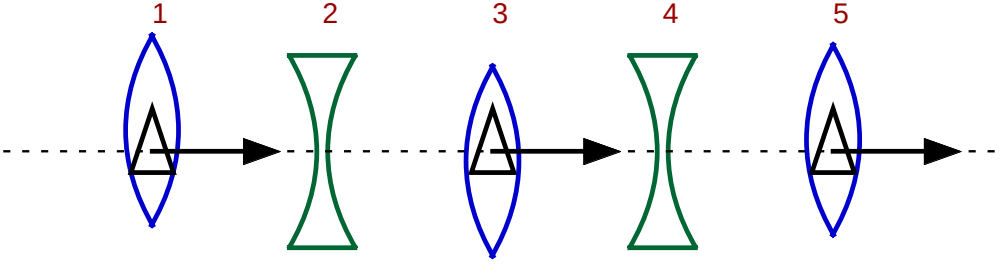

a
A
b
B
c
C
d
D
e
E
1.0
0.9
0.8
1
0
−1
polarization
100
200
300
400
500
Gγ


<S_y> at 8 o'clock vs Gγ
arc_quad&sext(σ_x,y)=1mm, arc_dip(σ_ψ)=1mrad,ε_x,y=20πmm.mrad
<S_y> dir. cosine
Gγ
<S_y> 180 part. surface
<S_y> 180 part. volume
CO ν_x=28.1930 ν_y=29.1850
0
0.2
0.4
0.6
0.8
1
415
420
425
430

(a) Ideal FODO

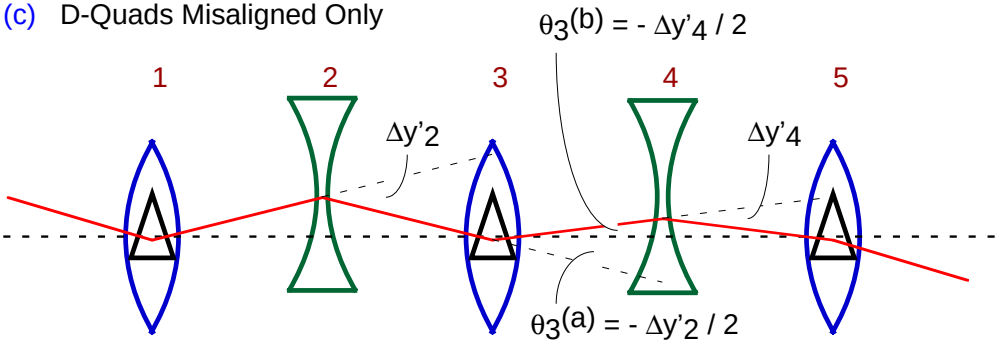


(b) F-Quads Misaligned Only



$\theta_1 = -d_1/F$ $\theta_3 = -d_3/F$ $\theta_5 = -d_5/F$

(c) D-Quads Misaligned Only



Closed Solution with Arc Orbit Correctors

$$\Theta_3^{(a)} = -\frac{1}{2}\Delta y_2' = -\frac{1}{2}\left(\frac{y_2 - d_2}{F}\right) = \frac{1}{2}\left[\frac{d_2}{F} - \frac{L}{F}\Theta_3^{(a)}\right]$$

which implies

$$\Theta_3^{(a)} = \left(\frac{1}{1+\frac{L}{2F}}\right)\frac{d_2}{2F}$$

and, likewise,

$$\Theta_3^{(b)} = \left(\frac{1}{1+\frac{L}{2F}}\right)\frac{d_4}{2F}.$$

Total deflection:

$$\Theta_3 = -\frac{d_3}{F} + \frac{1}{1+\sin(\mu/2)}\left(\frac{d_2+d_4}{2F}\right)$$

where μ is the phase advance per complete FODO cell.

Expected Corrector Strength

$$\begin{aligned}\Theta_{rms} &= \frac{d_{rms}}{F}\sqrt{1+\frac{1}{2}\left(\frac{1}{1+\sin(\mu/2)}\right)^2} \\ &\approx (1.08)\frac{d_{rms}}{F} \qquad (90^\circ \text{ FODO})\end{aligned}$$

The typical "residual" angle for a corrector:

$$\begin{aligned}\Theta_{rms}^{res} &= \left(\Theta_3 + \frac{d_3}{F}\right)_{rms} \\ &= \frac{1}{1+\sin(\mu/2)}\left(\frac{d_2+d_4}{2F}\right)_{rms} \\ &= \left(\frac{1}{1+\sin(\mu/2)}\right)\frac{\sqrt{2}}{2}\frac{d_{rms}}{F}\end{aligned}$$

Expected Residual Closed Orbit

Typical displacement at defocusing quadrupoles:

$$\begin{aligned}\check{y}_{rms} &= L\Theta^{(a)}_{rms} = L\Theta^{(b)}_{rms} = \frac{\sin(\mu/2)}{1+\sin(\mu/2)}\, d_{rms} \\ &\approx 0.41\, d_{rms} \qquad (90^\circ \text{ FODO})\end{aligned}$$

Residual closed orbit after "perfect" correction at the BPM's:

$$y_{rms} = \sqrt{\frac{\hat{y}^2_{rms} + \check{y}^2_{rms}}{2}} = \sqrt{\frac{0 + (0.41)^2 d^2_{rms}}{2}} = 0.29\, d_{rms}.$$

Thus, magnet alignment errors of rms 0.7 mm which are corrected by the arc dipole correctors $\Longleftrightarrow$ effective rms orbit of 0.2 mm.

Note:

Closed orbit generated by misaligned quadrupoles alone, with no correction:

$$\begin{aligned}y_{rms} &= \sqrt{\frac{\hat{y}^2_{rms} + \check{y}^2_{rms}}{2}} \\ &= \frac{d_{rms}\langle\beta\rangle}{2\sqrt{2}|\sin\pi\nu|F}\sqrt{N_Q} \\ &\approx 30\ d_{rms} \qquad (\text{for RHIC})\end{aligned}$$

Effective Local Spin Precession

Misaligned quadrupoles alone:

$$G\gamma(\hat{\theta}+\check{\theta}) = G\gamma(\hat{y}/F - \check{y}/F)$$

and hence $G\gamma\theta_{rms} \Longrightarrow$

$$\theta_{rms} = \sqrt{\hat{\theta}^2_{rms} + \check{\theta}^2_{rms}} = \sqrt{\frac{(\hat{y}/F)^2_{rms} + (\check{y}/F)^2_{rms}}{2}} = \sqrt{2}\,\frac{y_{rms}}{F}.$$

Corrected, misaligned FODO cell:

$$\begin{aligned}
G\gamma(\hat{\theta}+\check{\theta}) &= G\gamma\left(\Theta_3^{res} + \Delta y_4'\right) \\
&= G\gamma\left(\frac{1}{1+\sin(\mu/2)}\frac{d_2+d_4}{2F} - \frac{d_4-y_4}{F}\right) \\
&= G\gamma\left(\frac{1}{1+\sin(\mu/2)}\frac{d_2+d_4}{2F} + \frac{L}{F}\frac{1}{1+\sin(\mu/2)}\frac{d_4}{2F} - \frac{d_4}{F}\right) \\
&= G\gamma\left(\frac{1}{2F}\right)\left(\frac{1}{1+\sin(\mu/2)}\right)(d_2-d_4)
\end{aligned}$$

and so $G\gamma\theta_{rms} \Longrightarrow$:

$$\theta_{rms} = \frac{\sqrt{2}}{2(1+\sin(\mu/2))}\frac{d_{rms}}{F} = (0.29)\sqrt{2}\frac{d_{rms}}{F} \qquad (90^\circ \text{ FODO})$$

Again, magnet alignment errors of rms 0.7 mm corrected by dipole correctors $\Longleftrightarrow$ effective rms orbit of 0.2 mm.

Note on Dipole Roll:

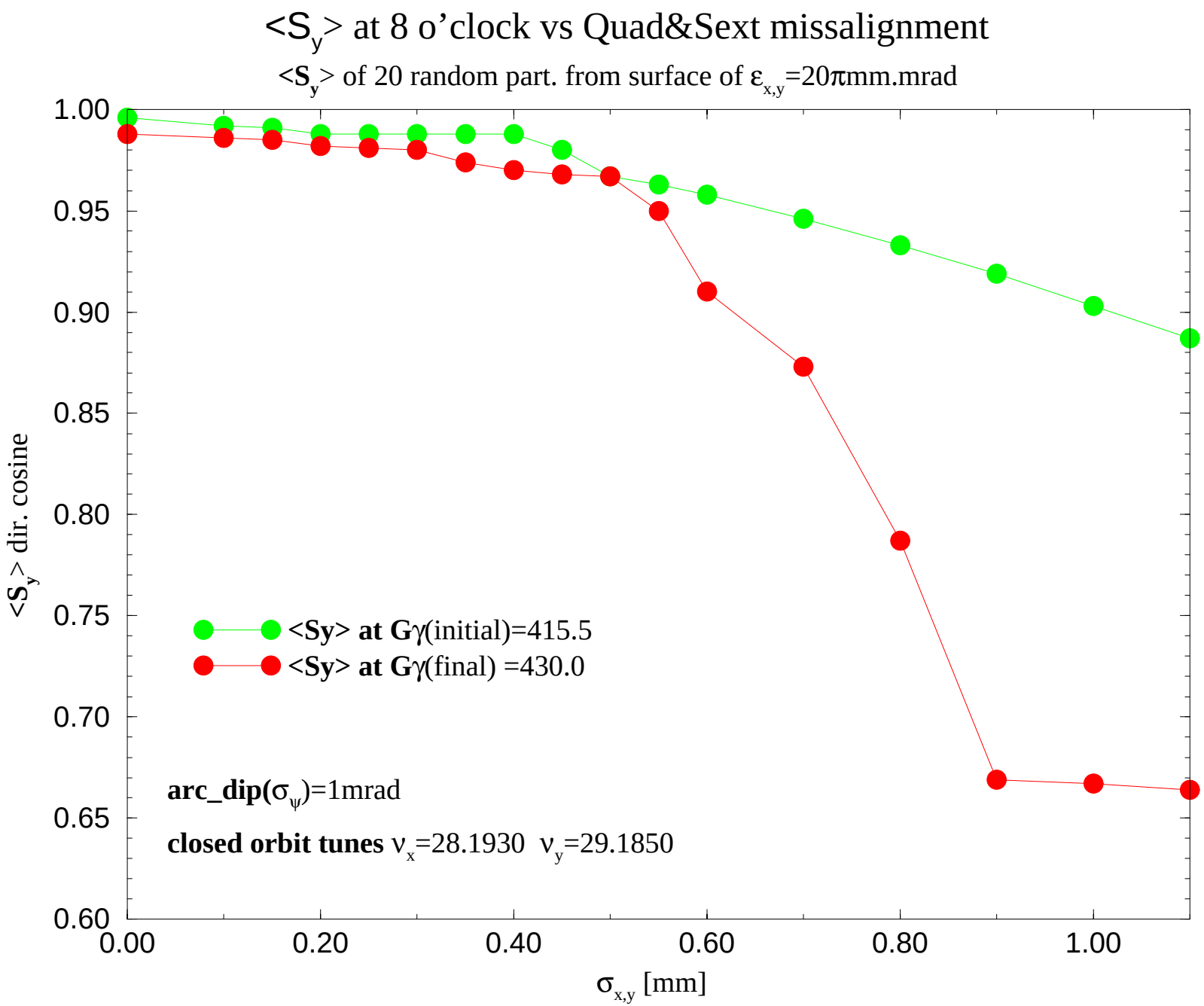
$$y_{rms} = \frac{\langle\beta\rangle}{2\sqrt{2}|\sin\pi\nu|}\sqrt{N_Q\left(\frac{d_{rms}}{F}\right)^2 + N_D\left(\frac{2\pi}{N_D}\right)^2\psi^2_{rms}}$$

implies

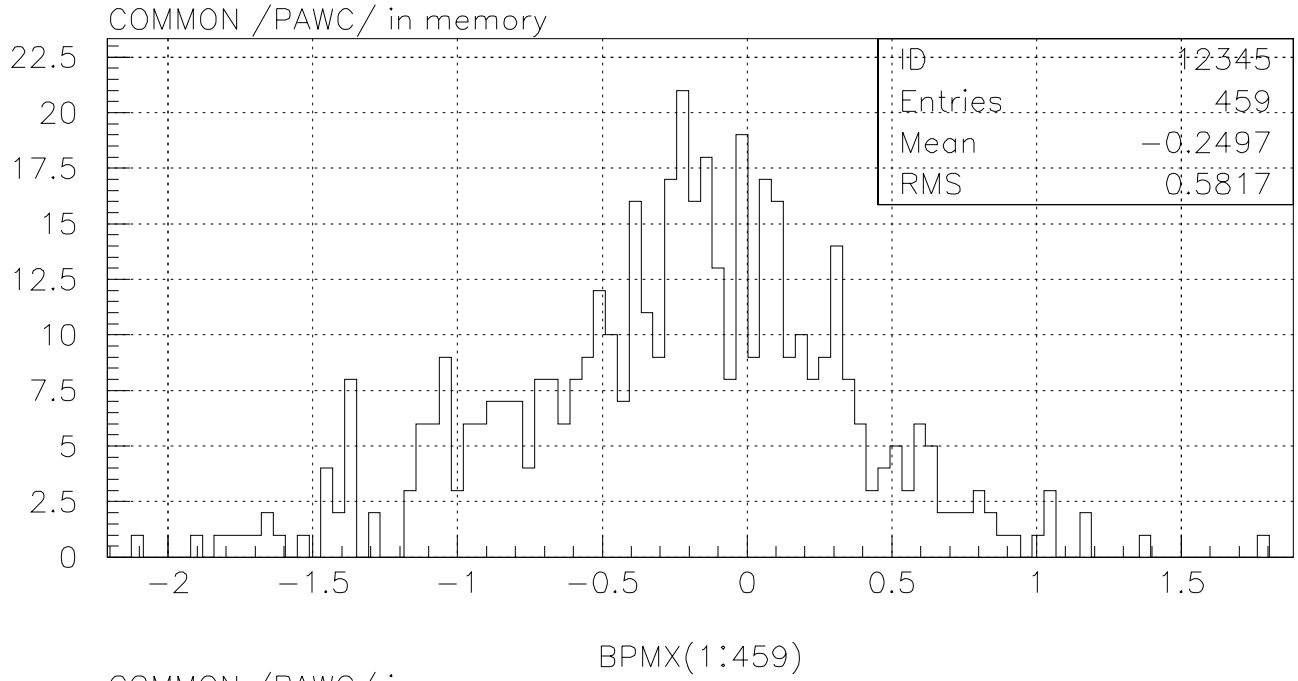
$$\psi_{rms} \approx \sqrt{N_D N_Q}\frac{d_{rms}}{2\pi F}$$

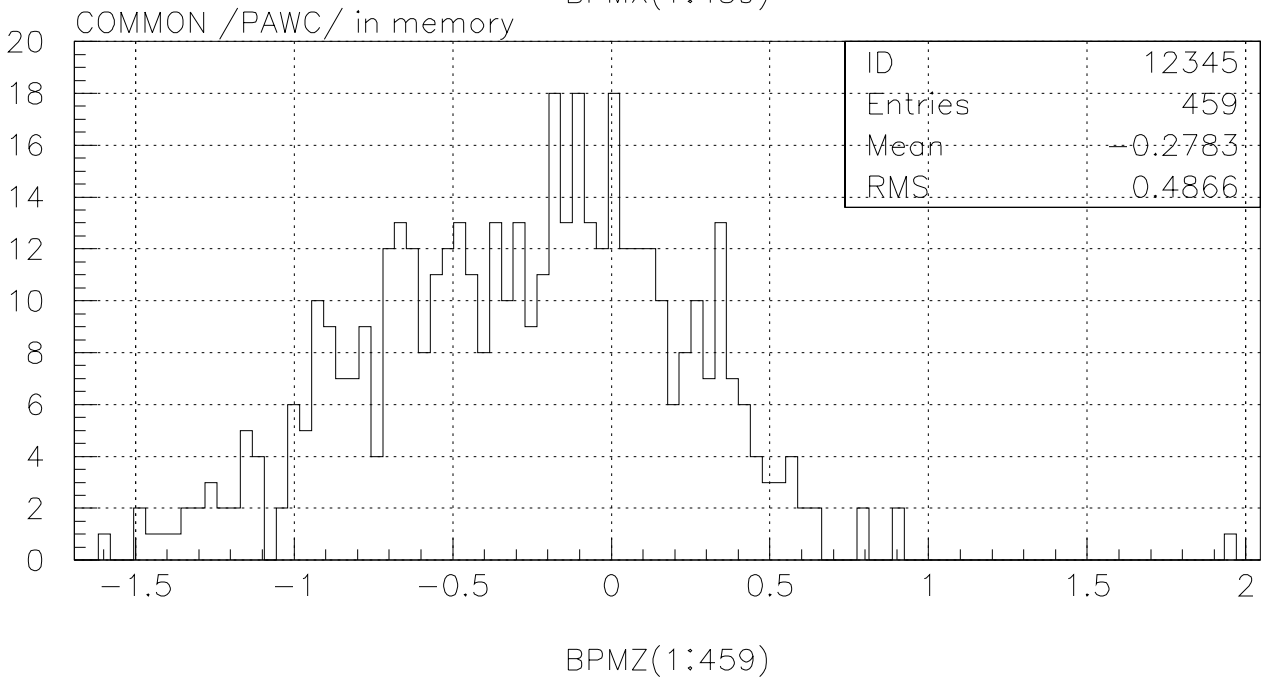
to play role in the dynamics.

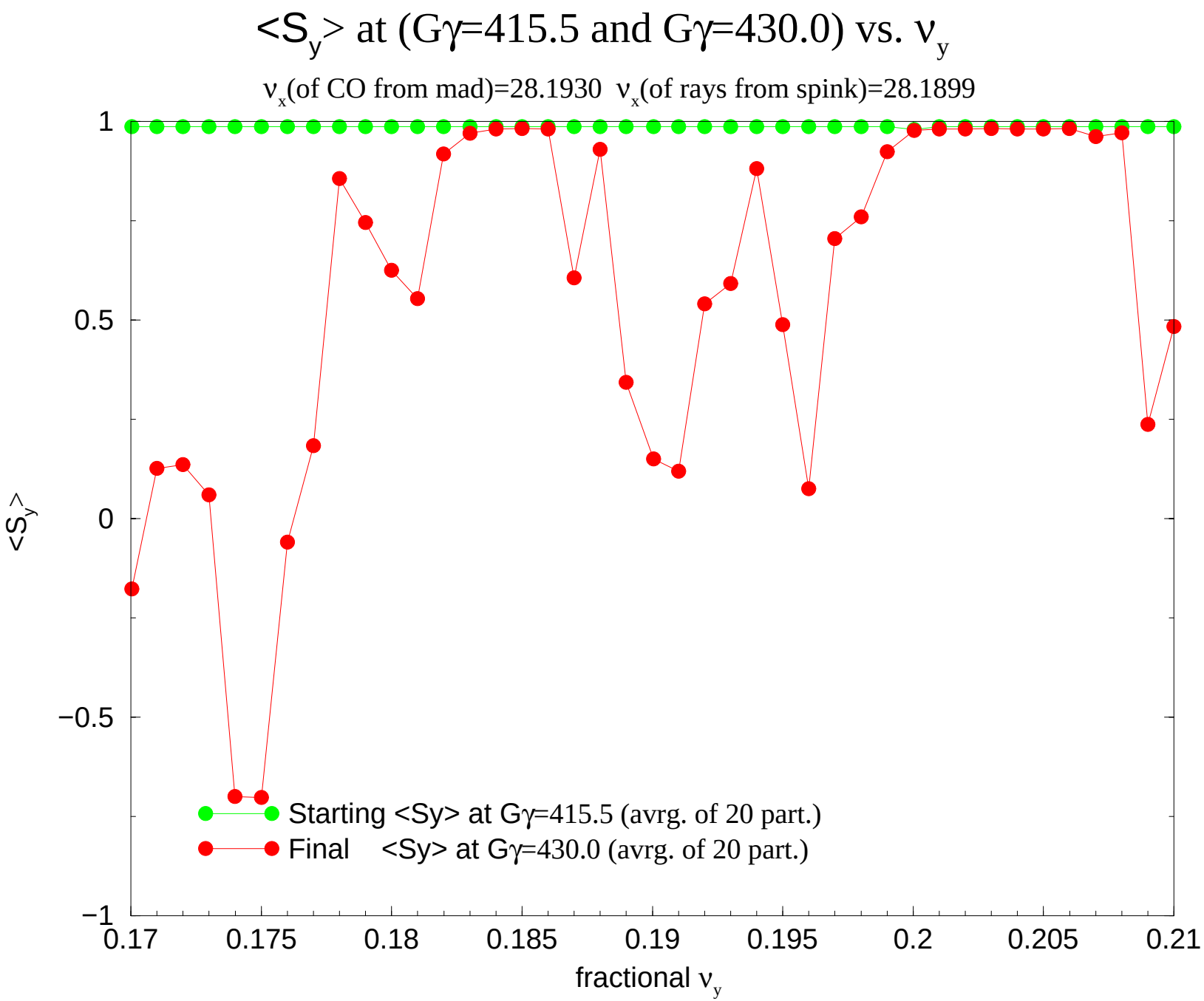
For RHIC, $\Longleftrightarrow$ $\psi_{rms} \approx 1.7$ mrad or larger.


<S_y> at 8 o'clock vs Quad&Sext missalignment
<S_y> of 20 random part. from surface of ε_x,y=20πmm.mrad
<S_y> dir. cosine
σ_x,y [mm]
<Sy> at Gγ(initial)=415.5
<Sy> at Gγ(final) =430.0
arc_dip(σ_ψ)=1mrad
closed orbit tunes ν_x=28.1930 ν_y=29.1850
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.00
0.20
0.40
0.60
0.80
1.00

98/05/28 17.35


COMMON /PAWC/ in memory
ID 12345
Entries 459
Mean −0.2497
RMS 0.5817
22.5
20
17.5
15
12.5
10
7.5
5
2.5
0
−2
−1.5
−1
−0.5
0
0.5
1
1.5
BPMX(1:459)


COMMON /PAWC/ in memory
ID 12345
Entries 459
Mean −0.2783
RMS 0.4866
20
18
16
14
12
10
8
6
4
2
0
−1.5
−1
−0.5
0
0.5
1
1.5
2
BPMZ(1:459)


<Sy> at (Gγ=415.5 and Gγ=430.0) vs. νy
νx(of CO from mad)=28.1930 νx(of rays from spink)=28.1899
1
0.5
0
−0.5
−1
<Sy>
0.17
0.175
0.18
0.185
0.19
0.195
0.2
0.205
0.21
fractional νy
Starting <Sy> at Gγ=415.5 (avrg. of 20 part.)
Final <Sy> at Gγ=430.0 (avrg. of 20 part.)