Fundamentals of Charged Particle Optics in High Energy Accelerator Systems

U.S. Particle Accelerator School
Winter 2008
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Class Overview

Students:

- 23 in the class
- ~15 from labs/research centers
- ~8 from universities; 1, high school
- Various stages/levels of education:
  - 3 PhD, 4 MS, 5 gs, 9 BS, 2 ug, 1 HS(!)
- credit (undergraduate) vs. audit

13 - Credit          10 - Audit

PLEASE CONFIRM -- initial sheet!
Course Overview

- Charged Particle Optics
- Beam transport (beam lines)
- Periodic (circular) systems
- Mostly, concerned with single particle effects

Why “high energy” in this course?

First week: basic, fundamental concepts

Second week: more details; design issues
Syllabus / Procedures

- lectures, labs, homework, exam
- lectures in mornings
- lecture/discussion + lab in afternoon
- study sessions in evenings

- physics vs. technology
- lots to cover in SHORT time !!
Homework/Labs

Many problems on the handout; not all to be assigned
Will choose from the list each day, ~4 problems
problems AND labs due 9:00 a.m. next morning
will go over HW in afternoon sessions
Enthusiasts can use ‘extra’ HW as practice; can go over during future discussion periods...
Hopefully Labs can be done in 2-hr slot; room also will be available at other times...
# Syllabus

## Week 1

<table>
<thead>
<tr>
<th>Day</th>
<th>9:00 - 10:15</th>
<th>10:30 - 11:45</th>
<th>1:30 - 2:45</th>
<th>3:00 - 5:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mon</td>
<td>Introduction &amp; Course Prerequisites</td>
<td>Single Particle Trajectories and Weak Focusing</td>
<td>Linear Guide Fields and Focusing Fields</td>
<td>Lab: Weak Focusing, Optical Elements</td>
</tr>
<tr>
<td>Tues</td>
<td>Matrix Formalism and Strong Focusing</td>
<td>Analytical Methods – Courant Snyder Parameters</td>
<td>Courant Snyder Parameters – II</td>
<td>Lab: Doublets, Triplets, Ray Tracing</td>
</tr>
<tr>
<td>Wed</td>
<td>Phase Space and Emittance</td>
<td>Off-Momentum Considerations</td>
<td>Homework Review &amp; Discussion</td>
<td>Lab: Lattice Parameters</td>
</tr>
<tr>
<td>Thur</td>
<td>Transverse Linear Errors and Adjustments</td>
<td>Additional Optics Components</td>
<td>Homework Review &amp; Discussion</td>
<td>Lab: Steering and Dispersion</td>
</tr>
<tr>
<td>Fri</td>
<td>Optical Design – cells and insertions</td>
<td>Optical Design – off-momentum</td>
<td>Homework Review &amp; Discussion</td>
<td></td>
</tr>
</tbody>
</table>

## Week 2

<table>
<thead>
<tr>
<th>Day</th>
<th>9:00 - 10:15</th>
<th>10:30 - 11:45</th>
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<th>3:00 - 5:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mon</td>
<td>Synchrotron Radiation</td>
<td>Light Source Lattices</td>
<td>Beam and Optics Diagnostics</td>
<td>Lab: Optical Insertions</td>
</tr>
<tr>
<td>Tues</td>
<td>Sensitivity Analyses</td>
<td>Beam Lines vs. Circular Accelerators</td>
<td>Homework Review &amp; Discussion</td>
<td>Lab: Beam Line Analysis</td>
</tr>
<tr>
<td>Wed</td>
<td>Consideration of Nonlinearities</td>
<td>Space Charge Effects</td>
<td>Homework Review &amp; Discussion</td>
<td>Lab: Synchrotron Analysis</td>
</tr>
<tr>
<td>Thur</td>
<td>Transverse Coupling</td>
<td>Emittance Exchange</td>
<td>Review Session</td>
<td>Lab: Finish Labs</td>
</tr>
<tr>
<td>Fri</td>
<td>Wrap Up 9:00-9:30 a.m.</td>
<td>Final Exam 10:00 a.m.-noon</td>
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</tbody>
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Today...

- Review of Classical Physics, esp. required concepts
  - Newton, Maxwell, special relativity, ...
- “Weak” Focusing synchrotron
- Linear Guiding and Focusing Magnetic Fields

Goals:
- review (get used to problem solving again!)
- get feel for techniques, range of parameters
Some Philosophy

- Optics design and calculations
- Modularity of Optical Systems
- Design Codes
- Acknowledgments
- Apologies

since from FNAL, SSC, BNL -- many examples are from these labs; will attempt to be general...
Homework for Tuesday

Problem Set 1 -- Nos. 3, 4, 5, 8, and 10
Invention of Particle Accelerators

- Early DC Accelerators -- van de Graaf, Cockcroft-Walton, ...
- AC Required for higher energies
  - Wideroe and recognition of RF
  - Invention of cyclotron by Lawrence, et al.
  - Radar leads to Alvarez linear accelerator
  - The Betatron principle
  - Invention of the Synchrotron -- McMillan, Veksler
  - Invention of Alternating Gradient Focusing
Modern Accelerators

- The HEP era -- SLAC, CESR, Tevatron, LEP, KEKb, PEP II, SSC, LHC, ...

- Also, modern-day Nuclear Physics -- CEBAF, RHIC

- Emergence of other interests -- medicine, defense, industry -- light sources, etc.

- Someone did a better job ...

- Where do those 1 Joule cosmic rays come from?
Single Particle Trajectories

- In each case -- single particle motion in magnetic fields
- Newton, Maxwell, Lorentz force, Relativity
- Magnetic rigidity
- The need for transverse focusing
  - “emittance” of a beam
  - space charge force
- stability of motion
- Electric vs. Magnetic forces on a charged particle
Special Relativity

- Frames of Reference
- The Principle of Relativity
- The Problem of the Velocity of Light
- Simultaneity
- Lengths and Clocks
- The Lorentz Transformation
- $E=mc^2$
- Transformations of E-, B-fields
Speed, Momentum, vs. Energy

Electron: 0 0.5 1.0 1.5 MeV
Proton: 0 1000 2000 3000 MeV
Single Particle Trajectories

- Newton, Maxwell, Lorentz force ✓
- Relativity ✓
- Magnetic rigidity
- The need for transverse focusing
  - “emittance” of a beam
- Space charge force
- Stability of motion
- Electric vs. Magnetic forces on a charged particle
Weak Focusing System

(as it has come to be known...)

- Field varies with radius:

\[ B = B_0 \left( \frac{R_0}{r} \right)^n \]

\[ n \approx \frac{R_0}{d} \]

\( n \) is determined by adjusting the opening angle between the poles

\( d = \infty, n = 0 \)

\( d = R_0, n = 1 \)
Weak Focusing System

Centr. Force:
\[
\frac{mv^2}{r} = e v B_0
\]

\[
B = B_0 \left( \frac{R_0}{r} \right)^n
\]

“field index”
\[
n \equiv - \frac{R_0}{B_0} \left( \frac{\partial B}{\partial r} \right)_{r=R_0}
\]

radial force

\[ r \]
\[ R_0 \]
\[ r \]
Weak Focusing System

- Differential Equations (Horizontal and Vertical)
- Betatron Tune
- Stability Condition
- Maximum Oscillation Amplitude
Weak Focusing: Differential Equations

Radial:

\[
\gamma m (\ddot{r} - r \dot{\theta}^2) = -evB_y = -evB_0 \left( 1 - n \cdot \frac{x}{R_0} \right)
\]

\[
\gamma m \ddot{r} = \gamma m \frac{v^2}{r} - evB_0 \left( 1 - n \cdot \frac{x}{R_0} \right)
\]

\[
\ddot{r} = \frac{v^2}{R_0} \left( 1 - \frac{x}{R_0} \right) - \frac{ev^2 B_0}{\gamma m v} \left( 1 - n \cdot \frac{x}{R_0} \right)
\]

\[
\dddot{x} = - \left( \frac{v}{R_0} \right)^2 (1-n)x
\]

\[
\dddot{x} + \omega_0^2 (1-n)x = 0
\]

- Betatron Tune
- # osc.'s per turn:

\[
\nu_x = \sqrt{1 - n}, \quad \nu_y = \sqrt{n}
\]
Maximum Excursions

- **Solution is Simple harmonic Oscillator:**

\[
\ddot{x} + \omega_0^2 \nu^2 x = 0 \\
v^2 x'' + \omega_0^2 \nu^2 x = 0 \\
x'' + \left(\frac{\nu}{R_0}\right)^2 x = 0
\]

\[\Rightarrow x(s) = x_0 \cos \left(\frac{\nu}{R_0} s\right) + x'_0 \frac{R_0}{\nu} \sin \left(\frac{\nu}{R_0} s\right)\]

\[x_{\text{max}} = \frac{R_0}{\nu} x'_0\]

- For given angular deflection, Maximum Excursion:
  - Note: 0 < tune < 1,
  - Thus, due to limited range of \( n \), then as \( R \) (i.e., energy) got large, so did the required apertures.
Desire particles “near” the design trajectory to remain near the design trajectory -- as we saw in weak focusing system, for small displacements, want a restoring force proportional to displacement.

Result: Simple Harmonic Motion

Will explore the use of linear fields

\[ B = \text{constant} \quad \text{or} \quad B = \text{constant} \times \text{displacement} \]
Guide Fields
and Linear Focusing Fields

\[ X(z) = \text{actual} \]

\[ X_d(z) = \text{design} \]

\[ x' = \frac{dx}{ds} \]

\[ s = \text{distance along ideal path} \]

\[ \gamma m \frac{d^2 X_d}{dt^2} = -ev_s B_0 \]
Linear Restoring Forces

- Assume linear guide fields: 
  \[ B_Y = B_0 + B'y \]
  \[ B_x = B'x \]

- Look at radial motion:

\[
\frac{dx}{dt} = \frac{dx}{ds} \frac{ds}{dt} = x'v_s
\]

\[
\frac{ds_1}{\rho + x} = \frac{ds}{\rho}
\]

\[
\frac{d\theta}{ds}
\]

\[
\frac{d\theta}{\rho}
\]

\[
\gamma m \frac{d^2(x_d)}{dt^2} = -e v_s B_0
\]

\[
\gamma m \frac{d^2(X_d + x)}{dt^2} = -e v_s B_y(X)
\]

\[
\gamma m (X''_d + x'') v_s^2 = -e v_s B_y(X)
\]

\[
\gamma m v_s x'' = -e \frac{v_s}{v_s} B_y + e B_0
\]

\[
\gamma m v_s x'' = -e \left[ B_y \left( 1 + \frac{x}{\rho} \right) - B_0 \right]
\]

\[
x'' = -\frac{e}{\rho} \left[ (B_y - B_0) + B_y \frac{x}{\rho} \right]
\]

\[
\approx -\frac{1}{B\rho} \left[ B'x + B_0 \frac{x}{\rho} \right]
\]
Hill’s Equation

Then, for vertical motion:

So we have, to lowest order,

\[ x'' + \left( \frac{B'}{B\rho} + \frac{1}{\rho^2} \right)x = 0 \]
\[ y'' - \left( \frac{B'}{B\rho} \right)y = 0 \]

General Form:

\[ x'' + K(s)x = 0 \]

\( (\text{SHO}, \text{for } K=\text{const.}) \)
Beam Line Components

- Electrostatic deflectors and the need to use magnetic fields
- Iron-dominated Magnetic Elements
  - dipole, quadrupole, n-pole magnets
  - combined function magnet
  - Lambertson magnets, solenoids, kickers, ...
- Hysteresis in iron magnets
- Solving Poisson’s Equation by “relaxation”
Magnetic Elements

\[ B_Y = B'x \]
\[ B_X = B'y \]
(Quadrupole Magnet)

\[ B_Y = B_0 \]
\[ B_X = 0 \]
(Dipole Magnet)
Gradient Magnets

\[ B_y = B_0 + B'x \]
\[ B_x = B'y \]

“Alternating Gradient”
Iron-dominated Magnets

Field Calculations of Simple Devices
  Dipole magnet
  Quadrupole magnet

More complex designs
Superconducting Magnets

- Here, field is not shaped by iron pole tips, but rather is shaped by placement of the conductor.
- Example: dipole magnet...

\[ 2\pi r B_\theta = \mu_0 J(\pi r^2) \]

\[ B_\theta = \frac{\mu_0 J}{2r} \]

\[ B_y = \frac{\mu_0 J}{2} d, \quad B_x = 0 \]

Current density \( J \)
Tevatron Dipole Magnet

“Cosine Theta” design
Superconducting Magnets

- Example:
  - $J \sim 1000 \text{ A/mm}^2$, $d \sim 1 \text{ cm}$
  - \[ B \sim \frac{1}{2}(4\pi \times 10^{-7})(1000 \times 10^6)(10^{-2}) = 2\pi \text{ Tesla} \]

- Tevatron -- \(~4.4 \text{ Tesla}\)
- SSC (above parameters) -- \(6.6 \text{ Tesla}\)
- LHC -- \(8 \text{ Tesla}\)
- LBNL model magnet -- \(16 \text{ Tesla}\)

Note: Higher fields a “plus,” but field quality typically easier to control with iron pole tips shaping the field …
“Relaxation” Method

- When the field is independent of z, the average potential on circle parallel to x-y plane equals the potential at the center of the circle, provided no charge density in the region.

- Generate “mesh” of points, with fixed potentials at the boundaries; generate “first guess” at values of potential at the mesh points.

- At each point, calculate average of potential of neighboring points; compare with the “guess”; alter the guess to split the difference.

- Repeat, until “converge” to a solution for which the average of the neighbors equals each “guess” value (to a certain accuracy). From the resulting potential, compute the field.
Sector Magnets

- Sector Dipole Magnet: “edge” of magnetic field is perpendicular to incoming/outgoing design trajectory:

  Field points “out of the page”
Sector Magnets & Sector Focusing

- Incoming ray displaced from ideal trajectory will experience more/less bending field, thus is “focused” toward axis in the bend plane:

Extra path length = $ds = d\theta \, x$
so extra bend angle = $dx = -ds/\rho$

$$dx = -(d\theta/\rho)x = -(1/\rho^2)x \, ds$$
or, $$x = -(1/\rho^2)x$$

Thus, $K_x = 1/\rho^2$, $K_y = 0$.
(as seen previously, with $B' = 0$)

For short magnet with small bend angle, acts like lens in the bend plane with

$$\frac{1}{f_x} = \frac{\theta}{\rho}$$
Edge Focusing

- In an ideal sector magnet, the magnetic field begins/ends exactly at $s = 0$, $L$ independent of transverse coordinates $x, y$ relative to the design trajectory.
  - *i.e.*, the face of the magnet is perpendicular to the design trajectory at entrance/exit
Edge Focusing

- However, could (and often do) have the faces at angles w.r.t. the design trajectory -- provides “edge focusing”

- Since our transverse coordinate $x$ is everywhere perpendicular to $s$, then a particle entering with an offset will see more/less bending at the interface...
- more later...
Some words on Space Charge

For most of this course, will neglect the force on a particle due to the presence of surrounding particles.

- Fields within a uniform “bunch”
- Fields within a Gaussian “bunch”

will say more about this next week.
Homework for Tuesday

Problem Set 1 -- Nos. 3, 4, 5, 8, and 10