De Sitter Days Workshop, 11/14/03 BR 1

Back Reaction of Cosmological Perturbations

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Outline: Introduction
Effective energy momentum tensor
Speculations & Criticisms
Local back-reaction

1. Introduction

Back-reaction

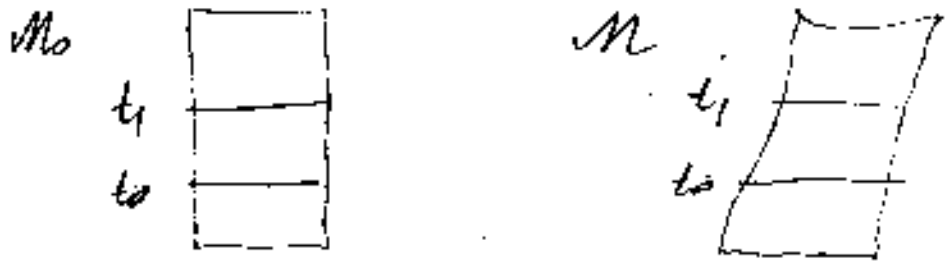
Cosmology $g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} \epsilon$
$\varphi = \varphi^{(0)}_{(1)} + \varphi^{(1)}_{(1)} \epsilon$

FRW eq. satisfy linearized pert. eq.

to $O(\epsilon^2)$ Einstein eq. not satisfied
in particular: homog. part of FE not satisfied

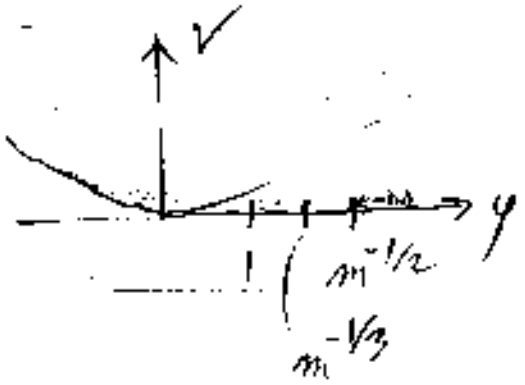
BR terms: terms which have to be added to background such that FE are satisfied to 2nd order

Physical Picture



$H(t)$ evolves differently in the presence and absence of fluctuations

Motivation: stochastic inflation



$V(\varphi) = \frac{1}{2} m^2 \varphi^2$

$1 < \varphi < m^{-1}$ inflation

$m^{-1/2} < \varphi < m^{-1}$ stochastic

$m^{-1/3} < \varphi$ BR important

History: 1960's Brill, Hartle, Isaacson
study BR of short wavelength gravit. waves

Our goal: Study BR of long wavelength cosmolog. pert.
scalar metric

2. Effective $T_{\mu\nu}$ $\qquad 8\pi G = 1$

$$G_{\mu\nu} = T_{\mu\nu}$$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \varepsilon g_{\mu\nu}^{(1)} + \varepsilon^2 g_{\mu\nu}^{(2)} = g_{\mu\nu}^{(0,br)} + \varepsilon g_{\mu\nu}^{(1)}$$

$$\varphi = \varphi^{(0)} + \varepsilon \varphi^{(1)} + \underbrace{\varepsilon^2 \varphi^{(2)}}_{\text{BR terms}} = \varphi^{(0,br)} + \varepsilon \varphi^{(1)}$$

homog. compt. only

insert into EE

solve for $g_{\mu\nu}^{(2)}$ homog. cpt. + spatial average

$$\langle G_{\mu\nu} \rangle = \langle T_{\mu\nu} \rangle$$

$$G_{\mu\nu}^{(0,br)} = T_{\mu\nu}^{(0,br)} + \tau_{\mu\nu} \qquad \text{BR eq.}$$

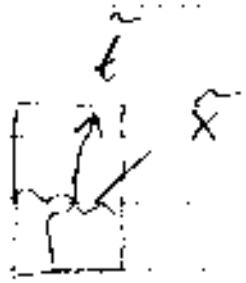
$$\tau_{\mu\nu} = \langle \underbrace{T_{\mu\nu}^{(2)} - G_{\mu\nu}^{(2)}}_{O(\varepsilon^2)} \rangle \qquad \text{eff. } T_{\mu\nu} \text{ of BR}$$

$O(\varepsilon^2)$ quadratic in $g_{\mu\nu}^{(1)}, \varphi^{(1)}$

Note: cumulative
scalar
small amplitude fluctuations can have large BR effect if large phase space of modes

Note: gauge problem

(Sketch)



Effective Energy-Momentum Tensor of Back-Reaction

$$ds^2 = (1+2\phi)\,dt^2 - a(t)^2(1-2\phi)\delta_{ij}\,dx^i dx^j$$

$$\varphi = \varphi_0 + \delta\varphi$$

$$\tau_{00} = 12H\langle\phi\dot{\phi}\rangle - 3\langle\dot{\phi}^2\rangle + 9a^{-2}\langle(\nabla\phi)^2\rangle + \frac{1}{2}\langle(\delta\dot{\varphi})^2\rangle + \frac{1}{2}a^{-2}\langle(\nabla\delta\varphi)^2\rangle + \frac{1}{2}V''\langle\delta\varphi^2\rangle + 2V'\langle\phi\,\delta\varphi\rangle$$

$$\begin{aligned}\tau_{ij} = a^{+2}\delta_{ij}\Big\{ & (24H^2 + 16\dot{H})\langle\phi^2\rangle + 24H\langle\dot{\phi}\phi\rangle \\ & + \langle(\dot{\phi}^2)\rangle + 4\langle\phi\ddot{\phi}\rangle - \frac{4}{3}a^{-2}\langle(\nabla\phi)^2\rangle \\ & + 4\dot{\varphi}_0^2\langle\phi^2\rangle + \frac{1}{2}\langle(\delta\dot{\varphi})^2\rangle \\ & - \frac{1}{6}a^{-2}\langle(\nabla\delta\varphi)^2\rangle - 4\dot{\varphi}_0\langle\delta\dot{\varphi}\phi\rangle \\ & - \frac{1}{2}V''\langle\delta\varphi^2\rangle + 2V'\langle\phi\,\delta\varphi\rangle\Big\}\end{aligned}$$

Result not surprising - need extra clock for physical argument of BR
CMB temp.

B Consider two matter field models

$$\mathcal{L} = \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - V(\varphi) + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi$$

$$u_\alpha = A\partial_\alpha\varphi \qquad A = (\partial^\mu\varphi\partial_\mu\varphi)^{-1/2}$$

In this case:

* valid for Φ

$$\frac{H}{\sqrt{1+2\Phi}} = \frac{1}{\sqrt{3}}\left[V(\varphi) + \frac{1}{2}\frac{\dot{\varphi}^2}{f_{00}} + \frac{1}{2}\frac{\dot{\chi}^2}{f_{00}}\right]^{1/2}$$

not the same with or without cosmological perturbations because ϕ enters f_{00}

consider evolution for a time (clock) interval δt ($\delta\chi$)

- slow rolling of φ
- linearization in δt

$$\Theta_f = \Theta_{in} - \frac{3(\chi_f - \chi_{in})}{\dot{\chi}_{in}\sqrt{f_{00}}}\left(\dot{\varphi}_{in}^2 + \dot{\chi}_{in}^2\right)$$

BR nonvanishing

" slows down expansion rate

Actually, we are not quite finished, because (xx) contains $\cdot$ which is not a derivative w.r.t. physical clock.

Further work: slow roll approximation

$$\Theta_f = \Theta_{in} - \frac{3}{2}\left(\frac{\Theta_{in}^2}{3} - V(\varphi_{in}, \chi_{in})\right)\frac{\delta\chi}{\chi'_{in}}$$

$\varphi_{in}(\phi \neq 0) \neq \varphi_{in}(\phi = 0)$ on a fixed χ_{in} surface

$\Downarrow$

BR effect

Future work:

* magnitude of ~~BR~~
* EOS of BR
* viability of scaling solution (observational)
* extension beyond pert. theory
* BR effects on fluctuations

Thm: BR eq is covariant under linearized diffeomorphisms.

Evaluation: scalar field matter $\varphi \to T_{\mu\nu}$

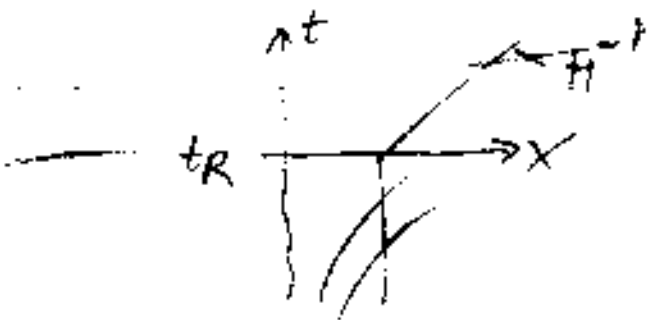
$$ds^2 = (1+2\phi)dt^2 - a(t)^2(1-2\phi)\delta_{ij}dx^i dx^j$$

$$\varphi = \varphi^{(0)} + \delta\varphi$$

$$\Downarrow$$

$T_{\mu\nu}$ slide

Application to Evolutionary Cosmology



IR phys. space grows
UV phys. space contract

ϕ and $\delta\varphi$ linked by Einstein constraint eq.

$$\dot\phi + H\phi = \frac{1}{2}\dot\varphi_0\,\delta\varphi$$

$$\Downarrow$$

all terms in $T_{\mu\nu}$ reduce to corr. fct. like

$$\langle\phi^2\rangle = \int dk\, k^2 |\phi_k|^2$$

theory of cosmology. pert. $\to \phi_k(t)$

almost constant on super Hubble scales

⇒ IR phase space dominates

focus on IR modes only

↓

all terms involving spatial & temporal gradients negligible

$$\rho_{br} = \left(2\frac{V''V^2}{V'^2} - 4V\right)\langle\phi^2\rangle$$

$$\boxed{\begin{aligned} p_{br} &= -\rho_{br} \\ \rho_{br} &< 0 \end{aligned}}$$

IR modes back-react like a negative cosmological constant

Magnitude: key: $|\rho_{br}(t)| \nearrow$ as $t \nearrow$

$$\frac{\rho_{br}(t)}{\rho_0(t)} \simeq \frac{3}{4\pi} m^2\varphi_0^2(t_i)\left(\frac{\varphi_0(t_i)}{\varphi_0(t)}\right)^4$$

If $\varphi_0(t_i) > \varphi_{cr} \sim m^{-1/3}$ then $\frac{\rho_{br}(t)}{\rho_0(t)} > 1$ by the end of inflation

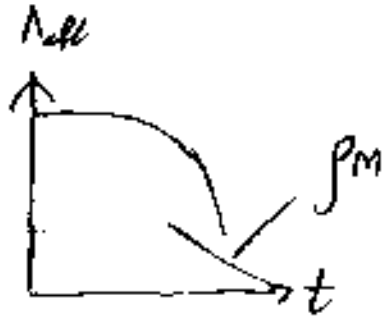
3. Generalizations / Caveats

Assume $g_{\mu\nu}$ and $g_{\mu\nu}^{(0)}$

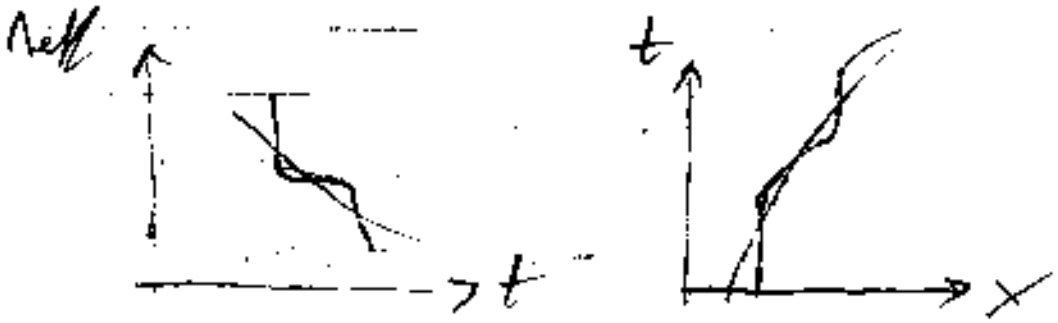
$$g_{\mu\nu}(t=0) = g_{\mu\nu}^{(0)} + g_{\mu\nu}^{br} + O(\varepsilon^4)$$

$$\Lambda_{eff} \qquad \Lambda_0 \qquad \rho_{br}$$

$$\Lambda_{eff}(t) = \Lambda_0 + \rho_{br}(t)$$



Seeking solution: $\Lambda_{eff}(t) \sim \rho_M(t)$
oscillations about this value



<u>Criticism</u> W. Unruh, A. Gott

- effect invariant under 2nd order gauge transf?
- $\langle \rho_{vac} \rangle$ is not measured
  is effect physically observable
- extension beyond perturbation theory?

<u>Why BR should be for real</u>

- BH argument
- cosmolog. pert. argument

4. Local Back-Reaction

A. Consider a simple fluid u^d velocity four vector

$$u^d u_d = 1$$

$$\Theta = u^d{}_{;d} \qquad H = \frac{1}{3}\Theta$$

Calculate Θ to second order in pert theory, IR terms only

$$(*) \qquad \Theta = 3\frac{\dot{a}}{a}\left(1 - \Phi + \frac{3}{2}\Phi^2\right) - 3\left(\dot{\Phi} + \dot{\Phi}\Phi\right)$$

at a fixed value of t

Problem t not observable

need to express Θ as a fn of a physical clock

Consider scalar field matter

$$u_d = A\partial_d\varphi \qquad A = \left(\partial^d\varphi\,\partial_d\varphi\right)^{-1/2}$$

find $H\left(1 - \Phi + \frac{3}{2}\Phi^2\right) = \frac{1}{\sqrt{3}}\sqrt{V(\varphi)}$

$$\Theta_{IR} = \sqrt{3}\sqrt{V(\varphi)}$$

$\Rightarrow$ dominant IR back reaction terms are physically measurable

N.B. $\dot{\Phi}$ terms from IR modes are locally measurable

$\nabla\Phi$ terms locally measurable