Oscillating bounce solutions & vacuum decay in de Sitter spacetime

J. Hackworth & EW

hep-th/0312???

Vacuum decay in flat spacetime

Vacuum decay in curved spacetime

"Oddities": decay of true vacuum

Hawking-Moss

Oscillating bounce solutions BANKS

Interpretation? JEOPARDY

Multidimensional WKB

Banks, Bender, Wu

$q_1, q_2 \cdots q_N$

$ds^2 = dq_1^2 + \cdots + dq_N^2$

$$0 = \delta \int ds \sqrt{2m(V-E)} \Rightarrow$$

Optimal tunneling path
Optimal exit point from barrier

$$\Updownarrow$$

$$0 = \delta S_{Euc} = \delta \int d\tau \left[\frac{m}{2} \left(\frac{dq_j}{d\tau} \right)^2 + V \right]$$

$q_j(\tau) \longleftrightarrow$ optimal path

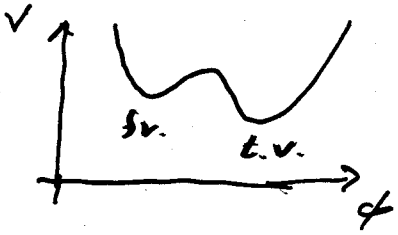
$q_j(\tau_f) \longleftrightarrow$ optimal exit pt.

$\tau_i \to \tau_f \to \tau_i \Leftrightarrow$ "Bounce"

rate $\sim e^{-B}$, $B = S_E(\text{bounce}) - S_E(\text{"false vac"})$

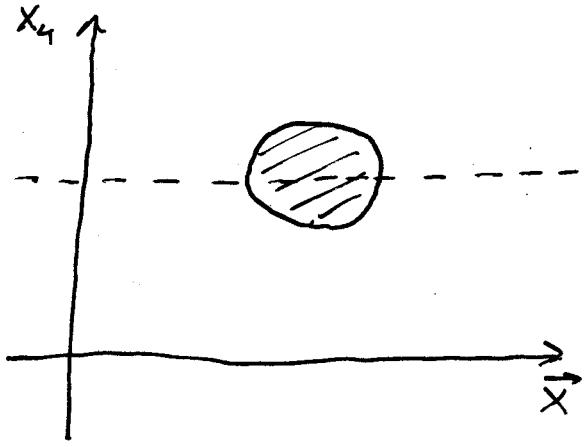
Field Theory

Coleman, Callan



$$\Box_E \phi = \frac{dV}{d\phi}$$

$$S_E = \int d^4 x_E \left[\frac{1}{2} (\partial_a \phi)^2 + V(\phi) \right]$$

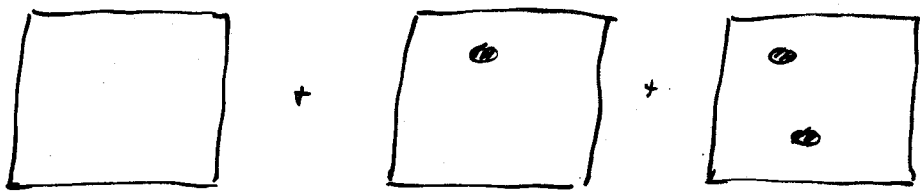


← initial data for classical evolution over-critical bubble

← initial state false vacuum

Prefactor

$$e^{-E_{fv}\mathcal{T}} = \int [d\phi]\, e^{-S_E[\phi]}$$

$=$  $+ \cdots$

$$\longrightarrow e^{-S_{fv}} \left[\det S''(\phi_{fv})\right]^{-1/2}$$

$$\longrightarrow e^{-S_{bounce}} \left[\underbrace{\det S''(\phi_b)}\right]^{-1/2}$$

$\longrightarrow$ 4 zero modes $\Rightarrow V\mathcal{T}$

$\longrightarrow$ 1 neg mode $\Rightarrow -i$

$$e^{-E_{fv}\mathcal{T}} = e^{-S_{fv}} \left(\det S''(\phi_{fv})\right)^{-1/2}$$
$$\times \left\{ 1 - iV\mathcal{T}Ke^{-B} + \tfrac{1}{2}\left(-iV\mathcal{T}Ke^{-B}\right)^2 + \cdots \right\}$$
$$= e^{-S_{fv}} \left(\det S''(\phi_{fv})\right)^{-1/2} e^{-iV\mathcal{T}\Gamma}$$

$$\frac{1}{V} \ln E_{fv} = \Gamma = Ke^{-B}$$

[Assume] SO(4) symmetry

$$\phi = \phi(s) \qquad s = \sqrt{\vec{x}^2 + x_4^2}$$

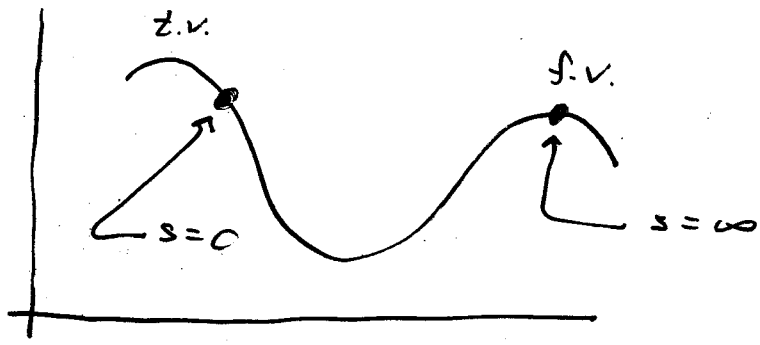
$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{dV}{ds}$$

$$\dot{\phi}(0) = 0 \,, \qquad \phi(\infty) = \phi_{fv}$$

Mechanical analogy:

Motion in potential $-V$

Friction $\sim 1/s$ $\Longrightarrow$ monotonically decreasing



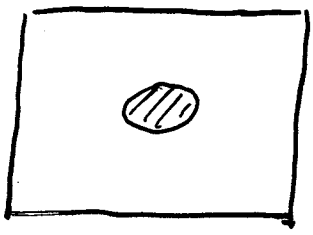
overshoot/undershoot argument

Finite temperature tunneling

$\ln E \to \ln F$

Euclidean path integral $\longrightarrow$ Periodic path integral

Period $= 1/T$



$+ \cdots$

Low T — Quantum fluctuations

$+ \cdots$

critical bubble = unstable static solutions

High T — Thermal fluctuations

Initial state not evident in "bounce"

Curved spacetime

Coleman : de Luccia

By analogy

$$\Gamma \sim e^{-B}, \qquad B = S_{Euc}(\text{bounce}) - S_{Euc}(\text{f.v.})$$

Assume $O(4)$ symmetry [Non-sym. solutions exist]

$$ds^2 = d\xi^2 + \rho^2(\xi)\left[d\chi^2 + \sin^2\chi \, d\Omega_2^2\right]$$

$$= d\xi^2 + \rho^2(\xi)\, d\Omega_3^2$$

$$\dot{\rho}^2 = 1 + \frac{8\pi}{3M_p^2}\left[\tfrac{1}{2}\dot{\phi}^2 - V\right]\rho^2$$

$$\ddot{\phi} + 3\frac{\dot{\rho}}{\rho}\dot{\phi} = \frac{dV}{d\phi}$$

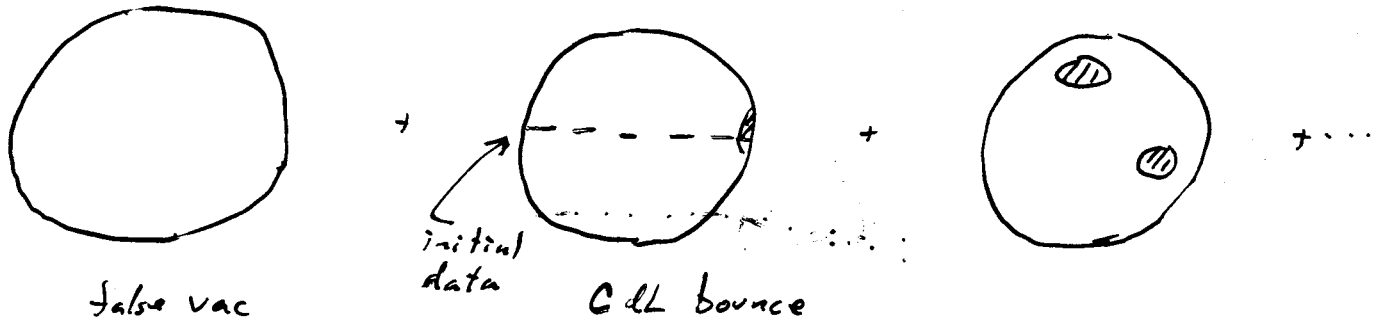
Consider $V(\phi) \geq 0$

$\Rightarrow$ $\rho(\xi)$ has two zeros $\xi = 0, \xi_{max}$ $\Longleftrightarrow$ S^4 topology

Boundary cond: $\dot{\phi}(0) = 0$ $\quad \dot{\phi}(\xi_{max}) = 0$

[Flat : $\dot{\phi}(0) = 0$ $\quad \phi(\infty) = \phi_{fv}$]

"Small bubbles" $m^{-1} \ll H^{-1}$

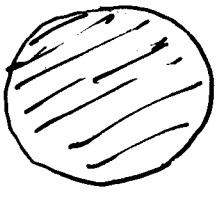


$$\phi = \phi_{fv} \Rightarrow \rho = H_f^{-1} \sin(H_f \xi)$$

$$S_E[f.v.] = -\frac{8\pi^2}{3}\frac{V_f}{H_f^{-4}} = -\frac{3}{8}\frac{M_p^4}{V_f}$$

$$S_E[\text{bounce}] - S_E[f.v.] \approx B_{flat}$$

Hawking-Moss solutions

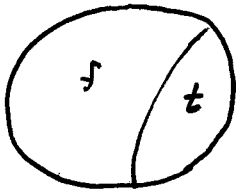


$$\phi \equiv \phi_{top}$$

$$S_{HM} = -\frac{3}{8}\frac{M_p^4}{V_{top}}$$

$$\Gamma \sim \exp -\left[\left(-\frac{3}{8}\frac{M_p^4}{V_{top}}\right) - \left(-\frac{3}{8}\frac{M_p^4}{V_{fv}}\right)\right]$$

Decay of true vacuum — K. Lee - EW



$$\Gamma_{f\to t} \sim \exp\left[-(S_{bounce} - S_{fv})\right]$$

$$\Gamma_{t\to f} \sim \exp\left[-(S_{bounce} - S_{tv})\right]$$

$$\frac{\Gamma_{t\to f}}{\Gamma_{f\to t}} = \exp\left[S_{tv} - S_{fv}\right] = \exp\left\{-\left[\frac{3}{8}\left(\frac{M_p^4}{V_{tv}} - \frac{M_p^4}{V_{fv}}\right)\right]\right.$$

Suppose $\Delta V = V_{fv} - V_{tv} \ll \bar{V} = \frac{1}{2}(V_{fv} + V_{tv})$

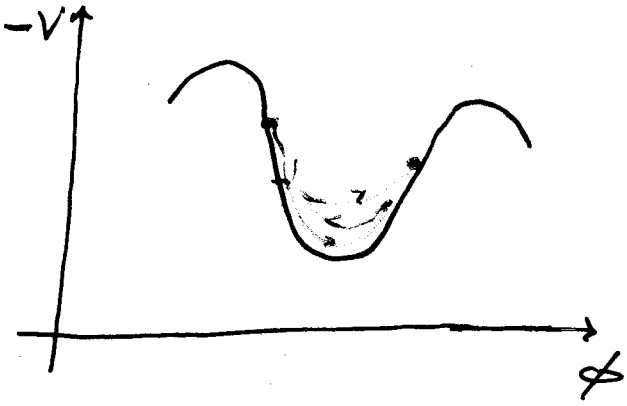
$$\simeq \exp\left\{-\frac{8\pi^2}{3} H^{-4}(\Delta V)\right\}$$

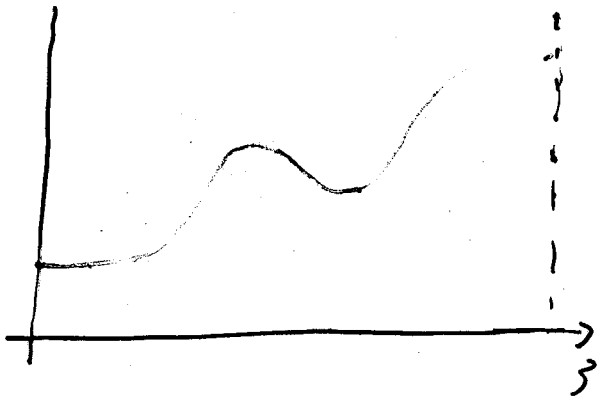
$$= \exp\left\{-\frac{4\pi}{3} H^{-3}(\Delta V)/T_H\right\}$$

Detailed balance $\Rightarrow$ $$\frac{v_{fv}}{v_{tv}} = \frac{\Gamma_{t\to f}}{\Gamma_{f\to t}} = e^{-(\Delta E)/T_H}$$

cf stochastic picture $\longleftrightarrow$ Starobinsky, Linde

Oscillating bounces?





Simplifications:

1) Assume SO(4) symmetry

2) Assume $(V_{top} - V_{min}) \ll V_{avg}$

$\Rightarrow$ To leading order, fix

$$\rho = H^{-1} \sin(H\xi)$$
$$= H^{-1} \sin y \qquad 0 \le y \equiv H\xi \le \pi$$

$\phi(y)$ obeys

$$\phi'' + 3 \cot y \, \phi' = \frac{1}{H^2} \frac{dV}{d\phi}$$

I) Small oscillations, $\phi \approx \phi_{top} \equiv 0$

$$\frac{dV}{d\phi} = -|V''(\phi_{top})|\,\phi + \cdots$$

$$0 \approx \phi'' + 3\cot y\ \phi' + \underbrace{\frac{|V''|}{H^2}}_{\beta}\ \phi$$

$$\phi = \frac{g(y)}{(\sin y)^{3/2}}$$

$$\Rightarrow\ 0 = g'' + \left[\left(\tfrac{3}{2}+\beta\right) - \tfrac{3}{4}\cot^2 y\right] g$$

$$\phi \sim A\,\frac{\sin\left[\sqrt{\beta+3/2}\ y + \delta\right]}{(\sin y)^{3/2}}\ , \quad \beta \gg \cot^2 y$$

$$\phi = A\, C_N^{3/2}[\cos y]$$

$$N(N+3) = \beta = \frac{|V''|}{H^2}$$

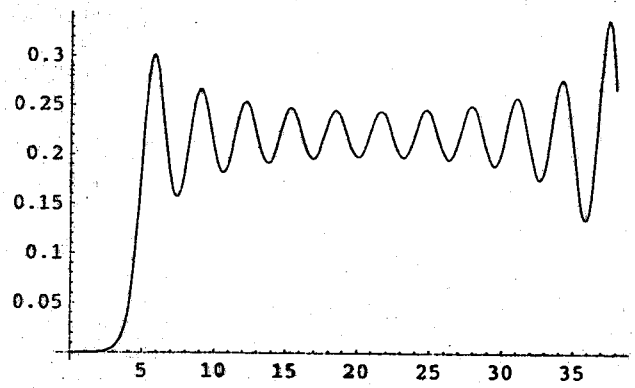
$\phi'(0) = 0$

$\phi'(\pi) = 0 \iff N =$ positive integer

A fixed by nonlinear terms

N odd: $fv \leftrightarrow tv$

N even: $fv \leftrightarrow fv$, $tv \leftrightarrow tv$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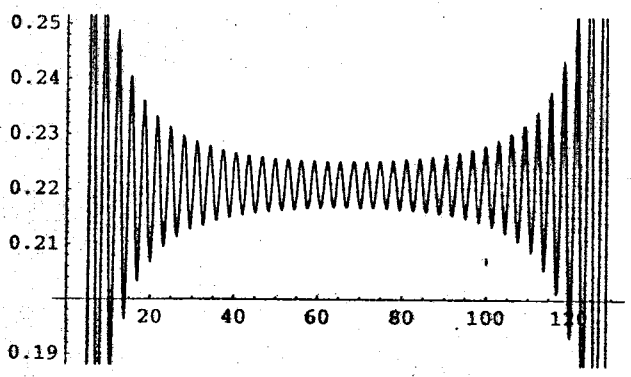
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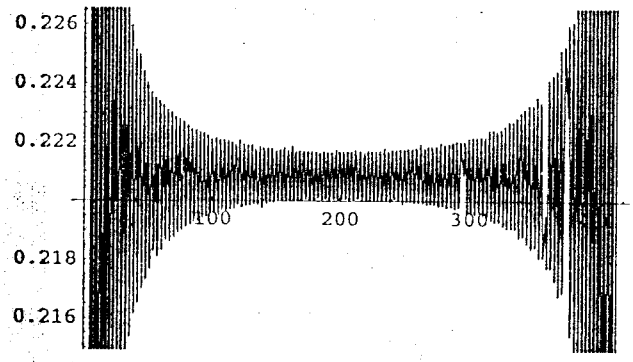
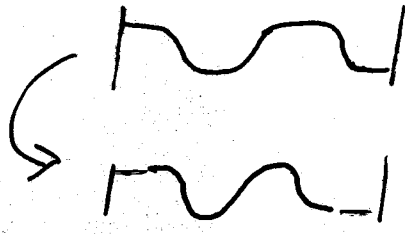
60
80
100
120


0.226
0.224
0.222
0.218
0.216
100
200
300

$$|V''| \to |V''| - \epsilon \iff N \to N - \delta$$

Solution disappears

$$|V''| \to |V''| + \epsilon \iff N \to N + \delta$$



Relative size of
$(-), (-) \iff V_{5v} - V_{tv}$

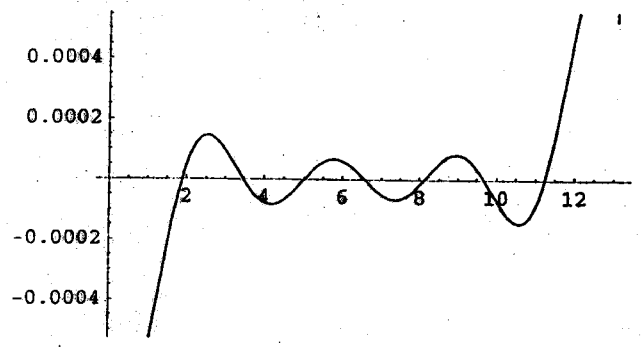
$|V''|$ increases $\iff N \to N+1$

New solution arises

$\frac{|V''|}{H^2} < 4$ Only H-M Jensen-Steinhardt

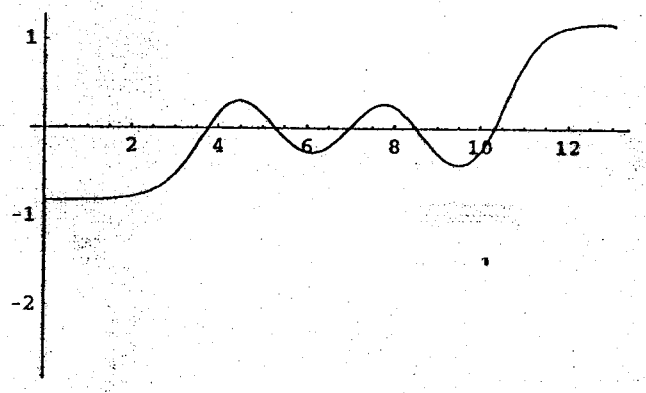
$4 < \frac{|V''|}{H^2} < 10$ H-M, CdL bounce

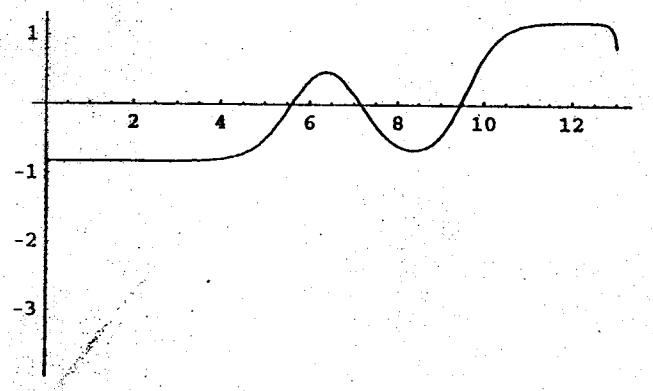
$N(N+3) < \frac{|V''|}{H^2} < (N+1)(N+4)$ H-M
CdL bounce
+ bounces with $1, 2, \cdots (N-1)$
intermediate max & min

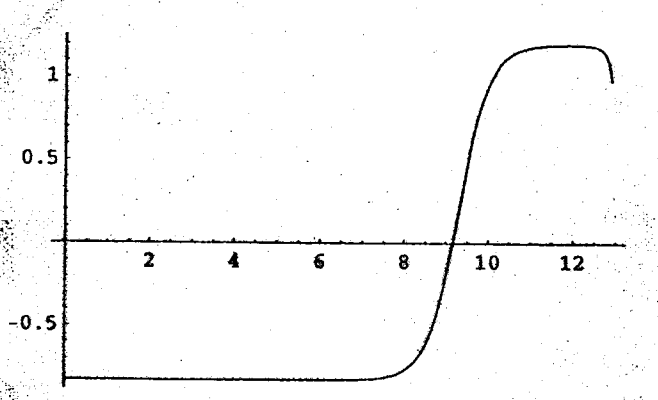


$v'' = -4$

$\Delta v = 1.05$

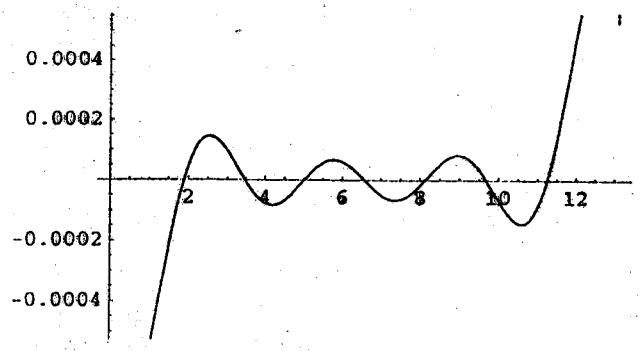






$v'' = -4$

$\Delta v = .2$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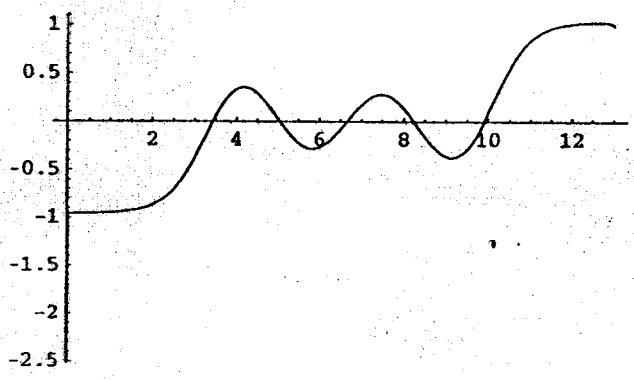
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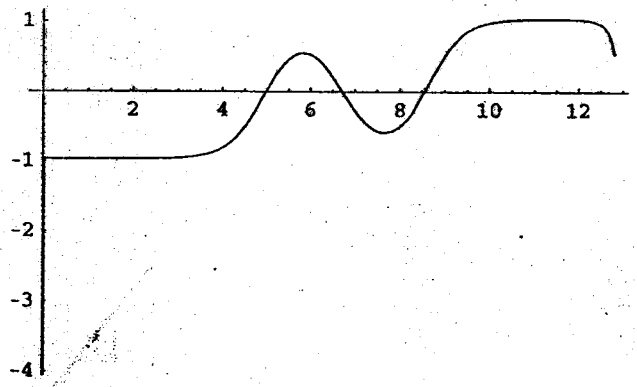
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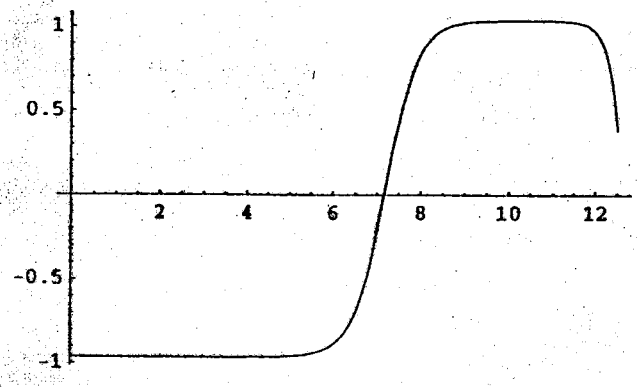
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Eigemodes about Hawking-Moss $\longleftrightarrow$ spherical harmonics on S^4

$$\lambda_N = N(N+3) - \frac{|V''|}{H^2}$$

$$\text{Degeneracy} = \frac{1}{6}(N+1)(N+2)(2N+3)$$

N	degeneracy	SO(4) - decomposition
0	1	1
1	5	1 + 4
2	14	1 + 4 + 9
3	30	1 + 4 + 9 + 16

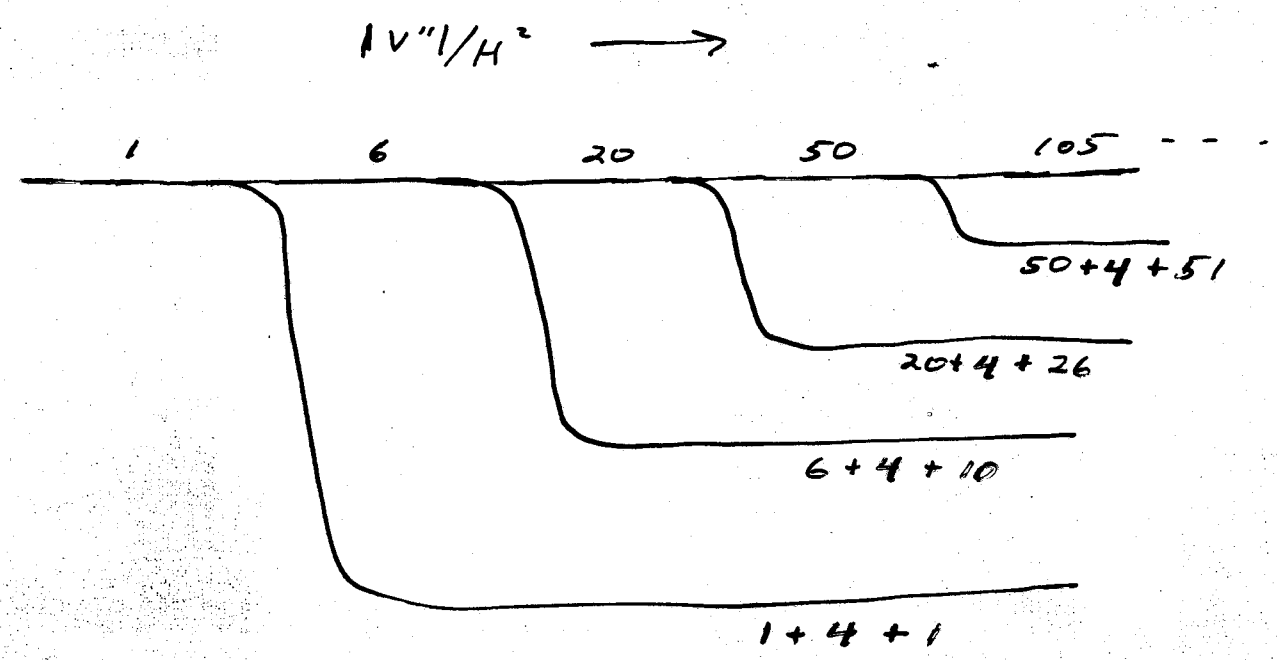
Negative modes: $N = 0, 1, \cdots N_* \approx \sqrt{\frac{|V''|}{H^2}}$

$$\#\text{ Neg. modes} = \sum_0^{N_*} \frac{1}{6}(N+1)(N+2)(2N+3)$$

$$\approx \frac{1}{12} N_*^4 \approx \frac{|V''|^2}{12H^4}$$

$$\frac{\text{Vol}[S^4]}{(\#\text{ neg. modes})} \approx \frac{32\pi^2}{|V''|^2} \sim \left(\frac{1}{\sqrt{|V''|}}\right)^4$$

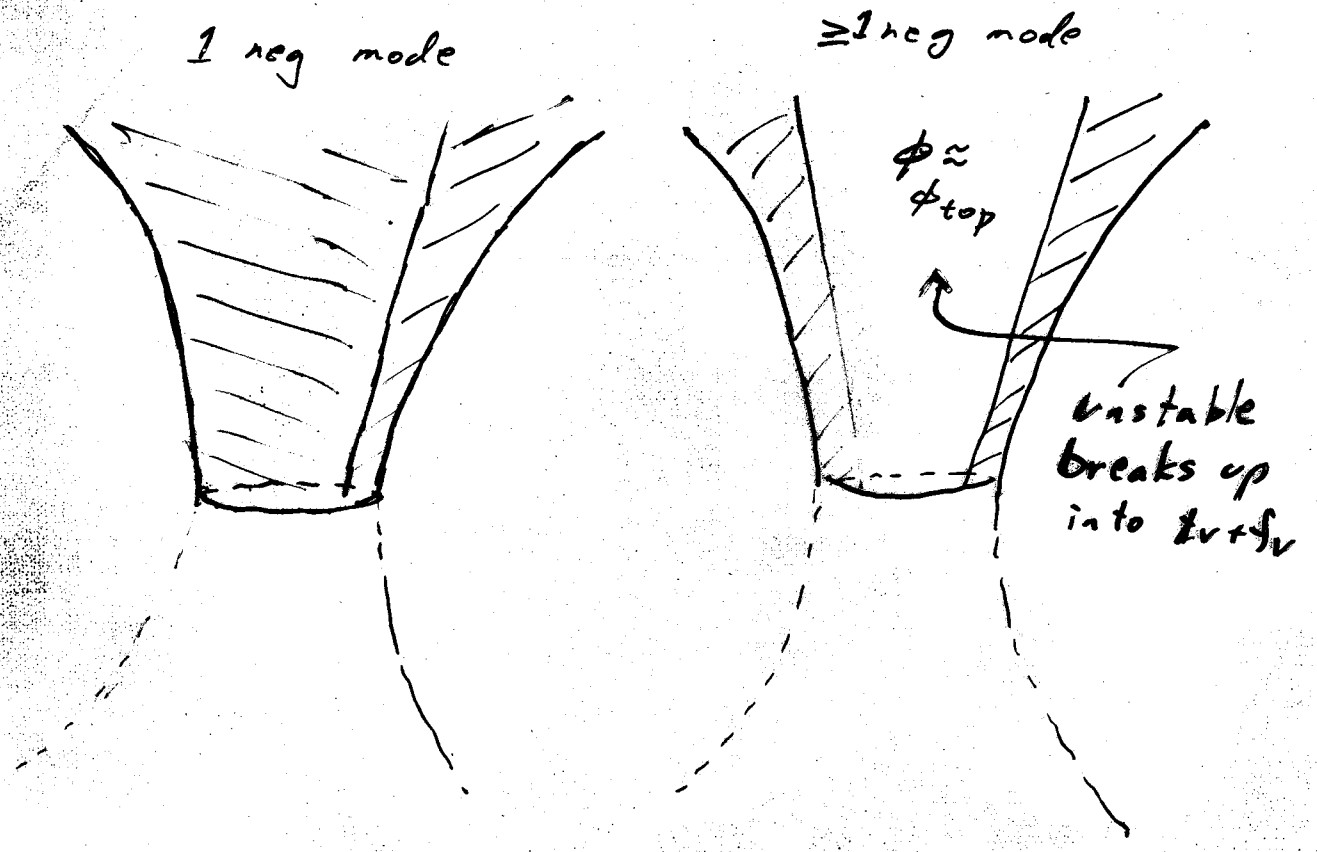
Bounce negative modes



– Negative modes.

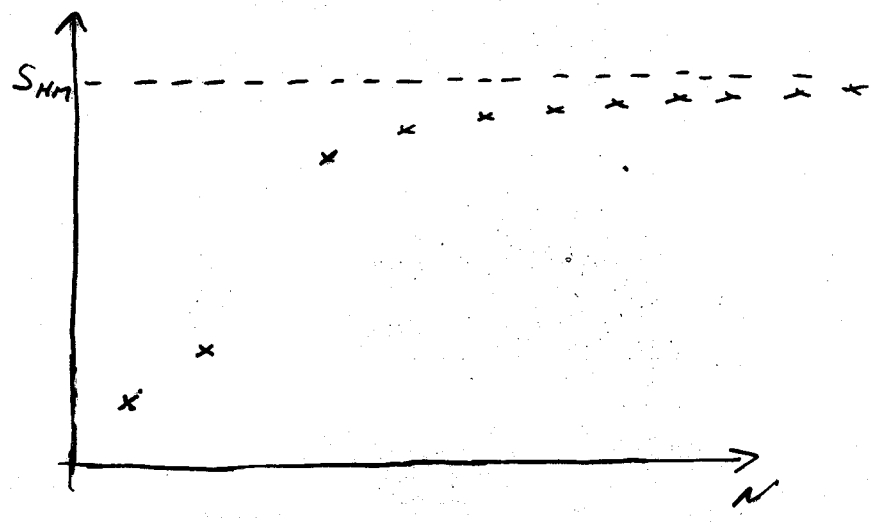
– Zero modes

– Pos/Neg? <u>Calculable</u>



Action

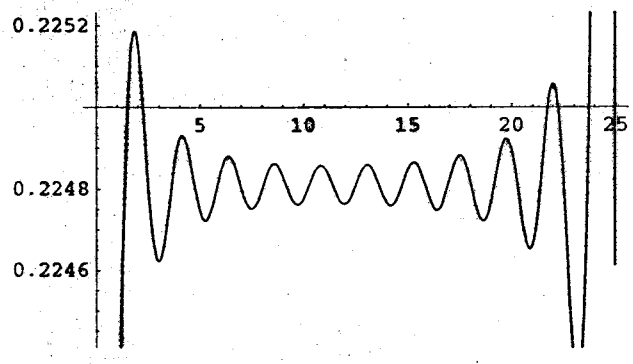
$\frac{|V''|}{H^2} >> 1$



$\frac{|V''|}{H^2} >> 1$ — CdL dominates
"Quantum tunneling" regime
deterministic real-time evolution

$\frac{|V''|}{H^2} \sim \mathcal{O}(1)$ — H-M, CdL, oscillating bounce
transition region

$\frac{|V''|}{H^2} << 1$ — H-M only
"Thermal tunneling" regime
stochastic real-time evolution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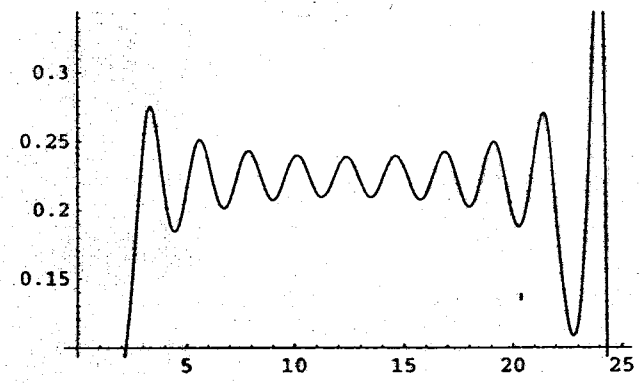
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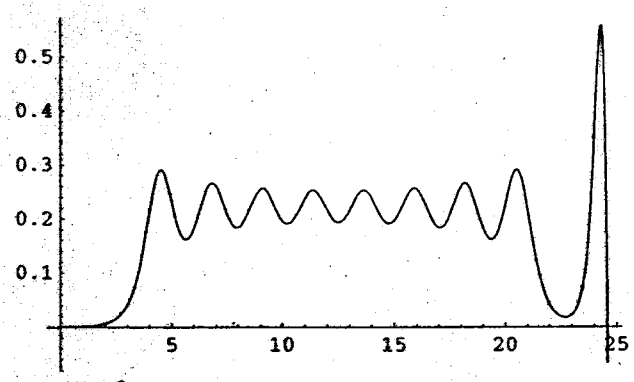
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$H = .239$ for $\Delta V = .2$, 1.05

$V'' = -4$

$\pi / H = 13.14$

$\pi \; 13.14 / \sqrt{2} = 9.29$