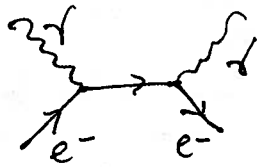
Homework Set #1 Solutions

(1) Lowest order Feynman diagrams

(i) Compton Scattering

(a)

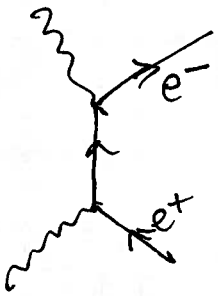


(b)

In (a), the electron emits a photon, the intermediate state is a virtual electron, then absorbs a photon, and appears as the outgoing electron.

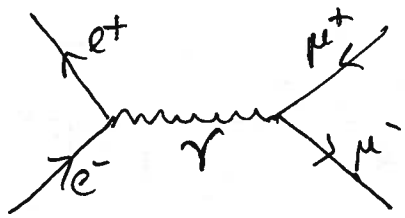
In (b), An electron absorbs a photon and re-emits it at a later time.

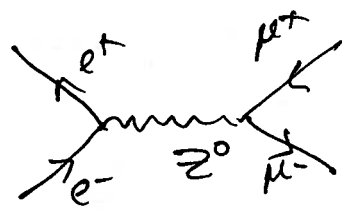
(ii) photon-photon interaction resulting in e^+e^- production.



Photon at the bottom goes into an e^+ and a virtual electron. The virtual e^- absorbs a photon and goes out as an electron.

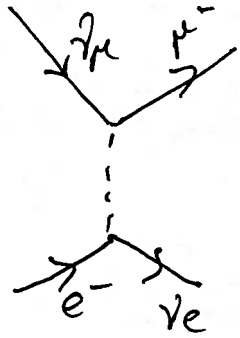
(iii) $e^+e^- \longrightarrow \mu^+\mu^-$





e^+ and e^- annihilate into a virtual photon or a virtual Z^0 boson that produce μ^+ and μ^-

(iv) $\nu_\mu e^- \longrightarrow \nu_e \mu^-$



An e^- emits a virtual W^- and changes into ν_e. The W^- absorbs ν_μ and turns into μ^-.

2. $A + B \rightarrow C_1 + C_2 + \cdots + C_n$

Condition for the reaction to take place,

$$E_{CM} \geq m_{C_1} + m_{C_2} + \cdots + m_{C_n}$$

$$E_{CM} = \sqrt{(E_A + E_B)^2 - (\vec{P}_A + \vec{P}_B)^2}$$

$$= \sqrt{m_A^2 + m_B^2 + 2(E_A E_B - \vec{P}_A \cdot \vec{P}_B)}$$

If B is at rest,

$$E_B = m_B, \quad |\vec{P}_B| = 0$$

$$E_{CM} = \left(m_A^2 + m_B^2 + 2 E_{th} m_B\right)^{1/2}$$

$$\therefore E_{th} = \frac{\left(\sum_i m_{C_i}\right)^2 - m_A^2 - m_B^2}{2 m_B}$$

3. Calculate threshold energy and beam momentum (Proton is at rest!)

(i) $\pi^- + p \rightarrow K^- + K^+ + n$

$$a + b \rightarrow c_1 + c_2 + c_3 + \cdots + c_n$$

$$E_{th} = \frac{\left(\sum m_{c_i}\right)^2 - m_a^2 - m_b^2}{2 m_b}$$

$$\therefore E_{th} = \frac{(2 m_{K^\pm} + m_n)^2 - m_\pi^2 - m_p^2}{2 * m_p}$$

$$= 1.5 \text{ GeV}$$

$$P_{beam} = \sqrt{E_{th}^2 - m_\pi^2} = 1.493 \text{ GeV}$$

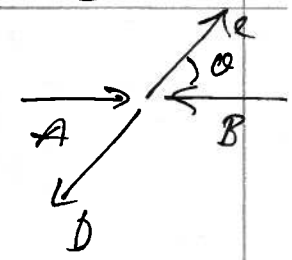
(ii) $p + p \rightarrow p + p + \pi^+ + \pi^-$

$$E_{th} = \frac{(2m_p + 2m_\pi)^2 - 2 m_p^2}{2 * m_p}$$

$$= 1.535 \text{ GeV}$$

$$P_{beam} = \sqrt{E_{th}^2 - m_p^2} = 1.215 \text{ GeV}.$$

4. Mandelstam variables for $A+B \to C+D$



$$s = (P_A + P_B)^2$$
$$= (E_A + E_B)^2 - (\vec{P_A} + \vec{P_B})^2$$
$$= m_A^2 + m_B^2 + 2E_A E_B - 2\vec{P_A}\cdot\vec{P_B}$$
$$= m_A^2 + m_B^2 + 2E_A E_B - 2|\vec{P_A}||\vec{P_B}|\cos\theta_{AB}$$

For elastic scattering of identical particles, in CM frame.

Set $m_B = m_A = m$; $E_B = E_A = E$, $|\vec{P_B}| = |\vec{P_A}| = p$

$$s = 2m^2 + 2E^2 + 2p^2$$
$$= 2m^2 + 2(m^2+p^2) + 2p^2$$
$$= 4(m^2+p^2)$$

$$t = (P_A - P_C)^2 = (E_A - E_C)^2 - (\vec{P_A} - \vec{P_C})^2$$
$$= 2m^2 - 2E^2 + 2p^2\cos\theta$$
$$= 2m^2 - 2(m^2+p^2) + 2p^2\cos\theta$$
$$= -2p^2(1-\cos\theta)$$

$$u = (P_A - P_B)^2 = (E_A - E_D)^2 - (\vec{P_A} - \vec{P_D})^2$$
$$= 2m^2 - 2(m^2+p^2) + 2p^2\cos(\pi+\theta)$$
$$= -2p^2(1+\cos\theta)$$

5. $p_{beam} = 400 \text{ GeV}/c$; $E_L = \sqrt{400^2 + .938^2}$

$= 400.0011$

$$E_{cm} = \sqrt{m_b^2 + m_t^2 + 2m_t E_L}$$

$$\simeq \sqrt{2 m_t E_L} \quad \text{if } E_L >> m_t, m_b$$

$$E_{cm} = \sqrt{2(400.0011)(.938)}$$

$$= 27.39 \text{ GeV}$$