Energy Loss of Electrons

Recall the Bethe-Bloch formula for heavy charged particles:

$$-\frac{dE}{dx} = K \cdot z^2 \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \gamma^2 \beta^2 \cdot E_{kin}^{max}}{I^2} - \beta^2 - \cancel{\frac{\delta}{2} - \frac{C}{Z}} \right]$$

ignore for now

For relativistic particles, $\beta \to 1$
with $E_{kin}^{max} = 2 m_e c^2 \gamma^2 \beta^2$,

$$-\frac{dE}{dx} \simeq K \cdot \frac{Z}{A} \cdot \left[\ln \frac{2 m_e c^2}{I} + 2 \ln \gamma - 1 \right]$$

When the incident particle is an electron, one cannot distinguish between the primary and the secondary electron after the collision.

$$E_{kin}^{max} = \frac{1}{2} (E - m_e c^2) \simeq \frac{1}{2} E \quad \text{for } E \gg m_e$$

$$-\frac{dE}{dx} = K \cdot \frac{Z}{A} \cdot \left[\ln \frac{2 m_e c^2}{I} + \frac{3}{2} \ln \gamma - \frac{1}{2} \ln 4 - 1 \right]$$

Bremsstrahlung ("Braking Radiation")

A charged particle accelerated or decelerated in the Coulomb field of a nucleus can emit a fraction of its energy as real photons.

The energy loss due to radiation can be calculated as:

$$-\left(\frac{dE}{dx}\right)_{rad} = N \int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E,\nu)\, d\nu$$

$$\boxed{-\left(\frac{dE}{dx}\right)_{rad} = 4\alpha N_A r_e^2 \cdot z^2 \cdot \frac{Z^2}{A} \cdot E \cdot \ln \frac{183}{Z^{1/3}}}$$

$\alpha = e^2/\hbar c$; $r_e^2 = e^2/mc^2$; $m =$ mass of the incident particle

To take into account interactions with electrons,

$$Z^2 \rightarrow Z^2 + Z = Z(Z+1)$$

For electrons, $z=1$, $m = m_e$, we write

$$-\frac{dE}{dx} = \frac{E}{X_0} \Leftarrow \text{This defines "Radiation Length}$$

$\therefore$ The Radiation Length of a material

$$X_0 = \frac{A}{4\alpha N_A Z(Z+1) r_e^2 \cdot \ln(183 Z^{-1/3})} \quad g/cm^2$$

Note: $X_0 \propto Z^{-2}$; $X_0 \propto m^2$.

For a mixture or a compound

$$X_0 = \frac{1}{\sum_{i=1}^{N} f_i / X_0^i}$$

; f_i = fraction of the ith component by weight

X_0^i = Radiation length of the ith Component.

$$-\frac{dE}{dx} = \frac{E}{X_0} \Rightarrow E = E_0 e^{-x/X_0} \quad (X_0(Fe) = 1.76 cm$$

Exponential attenuation of energy by radiation

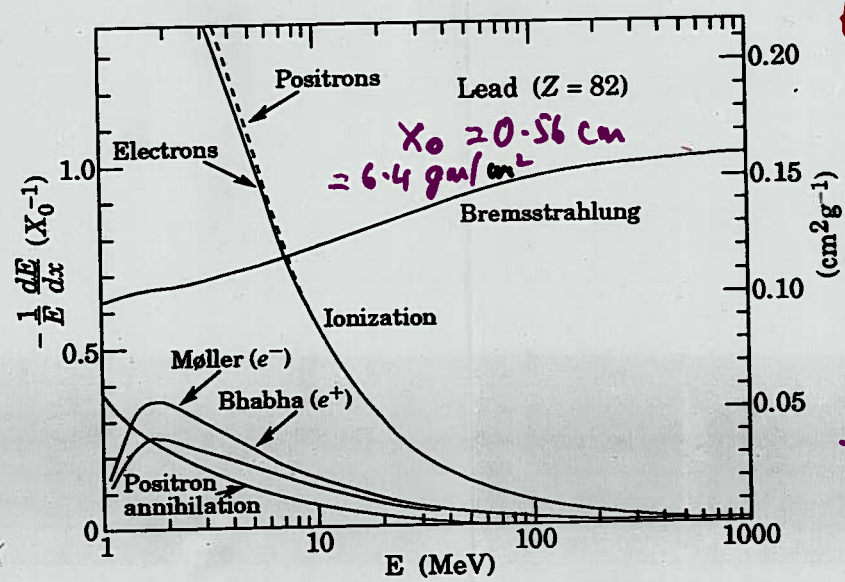
Compare with ionization loss:

$$\left.\frac{dE}{dx}\right|_{ion} \propto Z \quad ; \quad \left.\frac{dE}{dx}\right|_{rad} \propto Z^2$$

$$\left.\frac{dE}{dx}\right|_{ion} \propto \ln E \quad ; \quad \left.\frac{dE}{dx}\right|_{rad} \propto E$$

$(\gamma > 4)$

Material	Z	A	X_0 [g/cm^2]	X_0 [cm]	E_c [MeV]
Hydrogen	1	1.01	61.3	731 000	350
Helium	2	4.00	94	530 000	250
Lithium	3	6.94	83	156	180
Carbon	6	12.01	43	18.8	90
Nitrogen	7	14.01	38	30 500	85
Oxygen	8	16.00	34	24 000	75
Aluminium	13	26.98	24	8.9	40
Silicon	14	28.09	22	9.4	39
Iron	26	55.85	13.9	1.76	20.7
Copper	29	63.55	12.9	1.43	18.8
Silver	47	109.9	9.3	0.89	11.9
Tungsten	74	183.9	6.8	0.35	8.0
Lead	82	207.2	6.4	0.56	7.40
Air	7.3	14.4	37	30 000	84
SiO_2	11.2	21.7	27	12	57
Water	7.5	14.2	36	36	83



Energy where the two kinds of losses are equal is called the Critical Energy, E_c

$$-\frac{dE}{dx}(E_c)\Big|_{ion} = -\frac{dE}{dx}(E_c)\Big|_{brem}$$

Energy loss for electrons (positrons) by ionization and radiation and other mechanisms

For $Z \geq 13$,

$$E_c \simeq \frac{550 \text{ MeV}}{Z}$$

$$E_c^e(Pb) = 7.4 \text{ MeV} \quad ; \quad E_c^e(Fe) = 20.7 \text{ MeV}$$

$$E_c^\mu = E_c^e \cdot \left(\frac{m_\mu}{m_e}\right)^2 \quad ; \quad E_c^\mu(Fe) = 890 \text{ GeV}$$

For 100 GeV μ,

$$-\frac{dE}{dx} = \frac{E}{X_0} \cdot \left(\frac{m_e}{m_\mu}\right)^2 = 0.17 \text{ MeV}/\text{g/cm}^2$$

$$= 1.34 \text{ MeV/cm}$$

Direct Pair Production

At high energies, e^+e^- pairs can be produced by virtual photons in the Coulomb field of the nuclei.

e.g., $\mu + \text{nucleus} \rightarrow \mu + e^+ + e^- + \text{nucleus}$

$$-\left.\frac{dE}{dx}\right|_{pair} = b(Z, A, E) \cdot E$$

b varies only slightly with E

$$\therefore \left.-\frac{dE}{dx}\right|_{pair} \propto E$$

For 100 GeV μ, in Fe

$$-\left.\frac{dE}{dx}\right|_{pair} = 0.3 \ \frac{\text{MeV}}{\text{g/cm}^2} \qquad (= 2.36 \text{ MeV/}$$

Energy Loss by Photonuclear Interactions

Charged particles can interact inelastically via virtual photons with nuclei and lose energy

$$-\frac{dE}{dx}\Big|_{photonucl} = b_{nucl}(Z, A, E) \cdot E$$

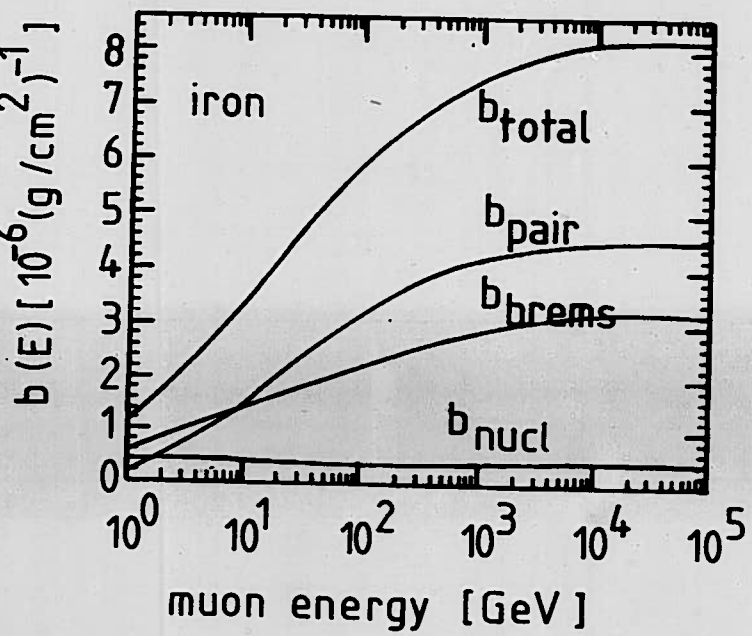
For 100 GeV muons in Fe,

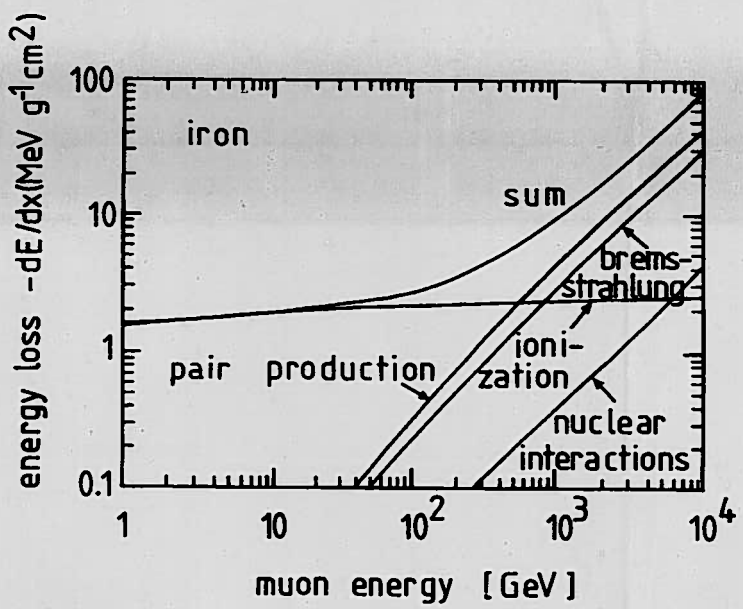
$$-\frac{dE}{dx}\Big|_{photonucl} = 0.04 \frac{MeV}{g/cm^2}$$

Total Energy Loss

$$-\frac{dE}{dx}\Big|_{total} = -\frac{dE}{dx}\Big|_{ion} - \frac{dE}{dx}\Big|_{brem} - \frac{dE}{dx}\Big|_{pair} - \frac{dE}{dx}\Big|_{photonucl}$$

$$= a(Z, A, E) + b(Z, A, E) \cdot E$$


iron
b_{total}
b_{pair}
b_{brems}
b_{nucl}
b (E) [10^{-6} (g /cm^2)$^{-1}$]
muon energy [GeV]


iron
sum
brems-
strahlung
ioni-
zation
pair production
nuclear
interactions
energy loss -dE/dx(MeV g^{-1}cm^2)
muon energy [GeV]

Interactions of Photons

Photons, unlike heavy charged particles, get attenuated in matter easily.

The number of photons lost from a monoenergetic beam of N photons, while passing through a thickness dx of a material.

$$dN = -\mu N dx \; ; \quad N = N_0 e^{-\mu x}$$

where μ = mass attenuation coefficient, and is related to the probability that a photon will be scattered or absorbed in the materia

$\therefore$ The intensity of a photon varies as

$$N = N_0 e^{-\mu x}$$

i.e., decrease exponentially with thickness of the material traversed.

Note: Contrast with energy attenuation of charged particles during radiation loss

Photoelectric Effect

- First explained by Einstein in 1905.
- An interaction between photon and the atom

$$\gamma + \text{atom} \rightarrow \text{atom}^+ + e^-$$

- Photons with energy $E_\gamma >> E_b$, the binding energy of an electron in the atom, may be absorbed and an atomic electron ejected with kinetic energy $T = E_\gamma - E_b$.
- "Resonant" absorption occurs with E_γ near the bound state energies of the electrons,

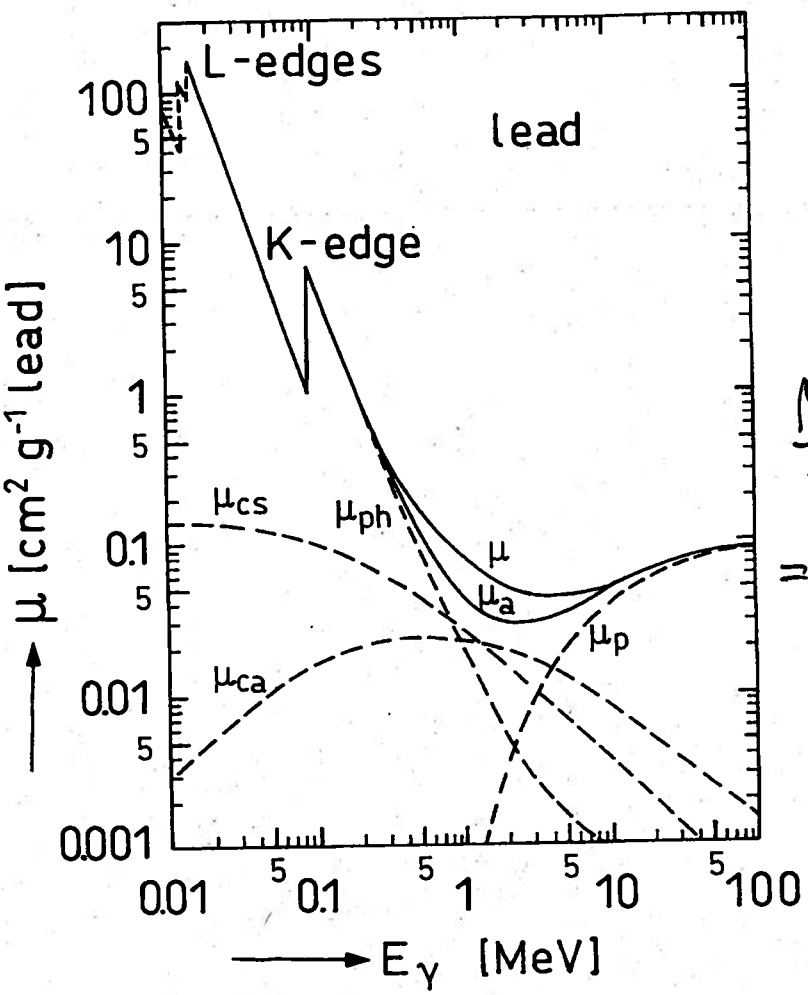
$$E_n = -13.6\,\text{eV} \cdot \frac{Z^2}{n^2} \qquad (n = \text{principal quantum number})$$

- The photoelectric cross section

$$\sigma_{pe} = \frac{32\pi}{3} \cdot \sqrt{2} \cdot Z^5 \alpha^4 \left(\frac{m}{\hbar\omega}\right)^{7/2} \cdot (\alpha\lambda\!\!\!^{-})^2$$

$$\sigma_{pe} \propto \frac{Z^5}{E_\gamma} \quad \text{for high energies}$$

- Main application in PMTs



$$I = I_0 e^{-\mu x}$$

$$\frac{\mu}{\rho} = \frac{N_A}{A} \cdot \sum_i \sigma_i$$

$$= \frac{N_A}{A} \left(\sigma_{pE} + Z\sigma_c + \sigma_{pai} \right.$$

$\frac{\mu}{\rho}$ = Mass attenuation coefficient

Fig. 1.14 d. Energy dependence of the mass attenuation coefficient μ and mass absorption coefficient μ_a for photons in lead [63, 73, 74, 75].

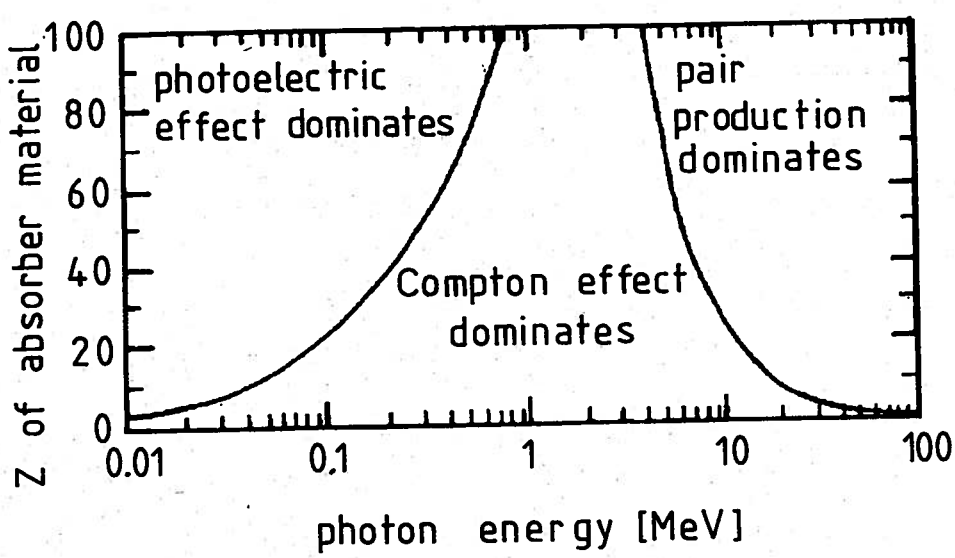


Fig. 1.15. Ranges, in which the photoelectric effect, Compton effect, and pair production dominate as a function of the photon energy and the target charge Z [37, 65, 68].

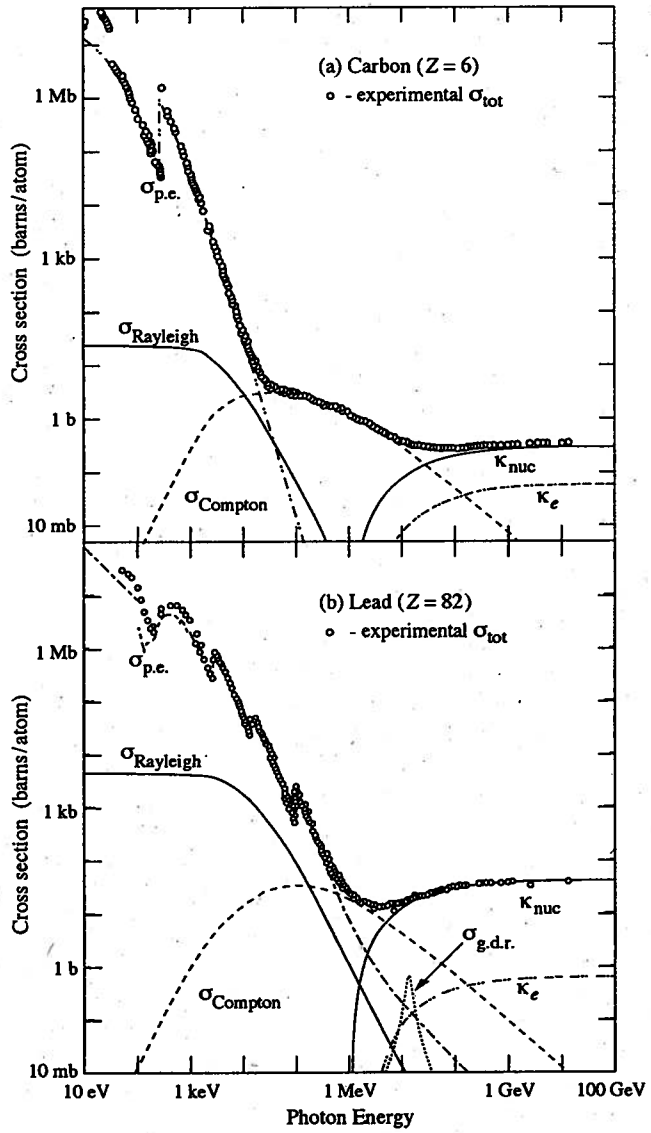


Figure 27.14: Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes:

$\sigma_{\text{p.e.}}$ = Atomic photoelectric effect (electron ejection, photon absorption)

σ_{Rayleigh} = Rayleigh (coherent) scattering–atom neither ionized nor excited

σ_{Compton} = Incoherent scattering (Compton scattering off an electron)

κ_{nuc} = Pair production, nuclear field

κ_e = Pair production, electron field

$\sigma_{\text{g.d.r.}}$ = Photonuclear interactions, most notably the Giant Dipole Resonance [46]. In these interactions, the target nucleus is broken up.

I

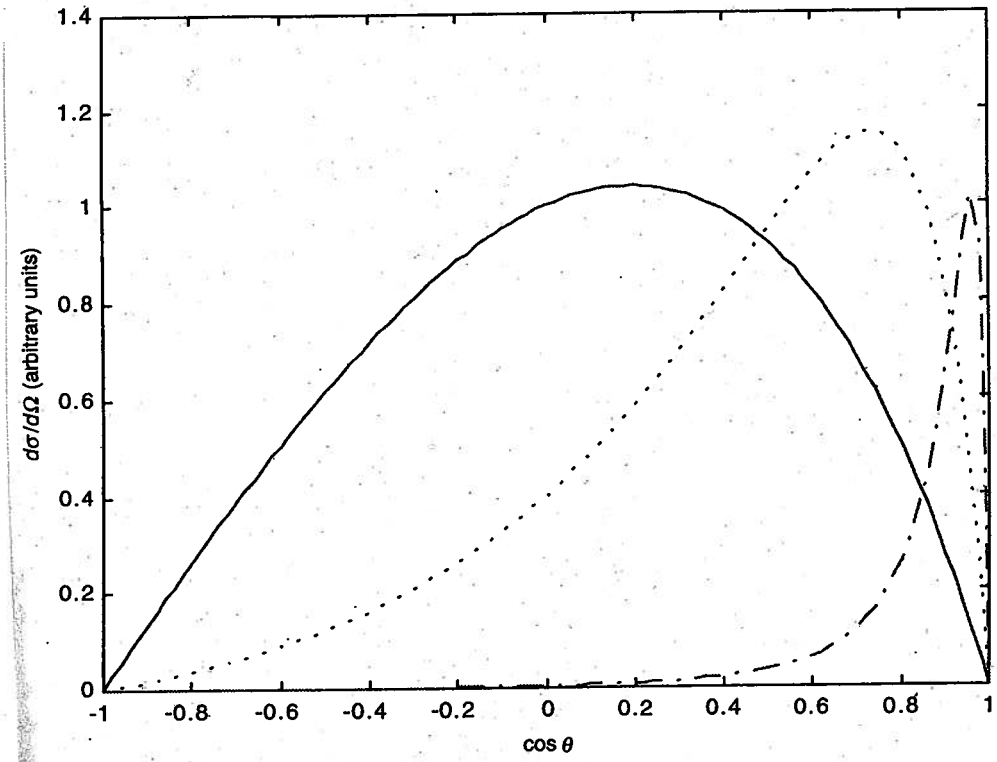
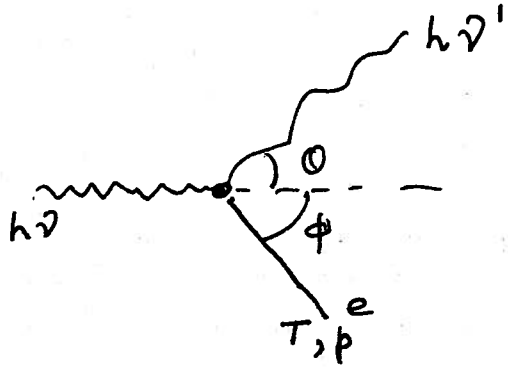


Fig. 2.6. Angular distribution of the photoelectron with respect to the incident photon direction for several recoil electron velocities. —, $\beta=0.1$; · · ·, $\beta=0.5$; –·–·, $\beta=0.9$.

Compton Effect

- Scattering of an incident photon off a quasi-free atomic electron.

$$\gamma + e \rightarrow \gamma' + e'$$



From energy momentum conservation,

$$h\nu = h\nu' + T$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p\cos\phi$$

$$0 = \frac{h\nu'}{c}\sin\theta - p\sin\phi$$

Using these and

$$T = \sqrt{p^2c^2 + m_e^2c^4} - m_ec^2,$$

One can derive a number of useful relations

The frequency of the scattered photon:

$$\nu' = \frac{\nu}{1+\epsilon(1-\cos\theta)} \quad \text{—— (1)} \qquad \epsilon = \frac{h\nu}{m_e c^2}$$

The kinetic energy of the recoil electron:

$$T = m_e c^2 \cdot \frac{\epsilon^2(1-\cos\theta)}{1+\epsilon(1-\cos\theta)} \quad \text{—— (2)}$$

The electron recoil angle:

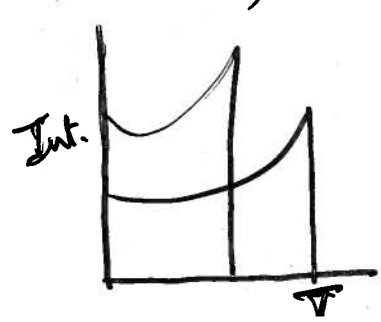
$$\cos\phi = (1+\epsilon)\left[\frac{1-\cos\theta}{2+\epsilon(\epsilon+2)(1-\cos\theta)}\right]^{1/2} \quad \text{—— (3)}$$

For backscattering $(\theta = \pi)$, (2) becomes,

$$T = T^{max} = \frac{2\epsilon^2}{1+2\epsilon} \cdot m_e c^2 = \frac{2\epsilon}{1+2\epsilon} \cdot E_\gamma$$

For $\epsilon \gg 1$, i.e., $E_\gamma = h\nu \gg m_e c^2$,

$$T^{max} \simeq E_\gamma$$



$$E_\gamma - \frac{m_e c^2}{2}$$

Compton Edge

The cross section for compton scattering can be calculated using QED.

Klein - Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \cdot \frac{1}{[1+\epsilon(1-\cos\theta)]^2} \left(1+\cos^2\theta + \frac{\epsilon^2(1-\cos\theta)^2}{1+\epsilon(1-\cos\theta)}\right)$$

$$\sigma_c = \int \frac{d\sigma}{d\Omega} \cdot d\Omega$$

(Probability/electron for a compton scattering to occur.)

$$\sigma_{cs} = \frac{E_\gamma'}{E_\gamma} \cdot \sigma_c$$

scattered cross section

$$\sigma_{ca} = \sigma_c - \sigma_{cs}$$

absorption cross section

Pair Production

Production of electron-positron pairs is possible in the coulomb field of a nucleus (or an atomic electron) if $E_\gamma > E_{th}$

From energy & momentum conservation, the threshold energy can be calculated.

$$E_\gamma \geq 2 m_e c^2 + 2 \cdot \frac{m_e^2}{m_{nucleus}} c^2$$

But $m_{nucleus} >> m_e$

$\therefore E_\gamma \geq 2 m_e c^2$ is the threshold

$$\sigma_{pair} = 4 \alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} \right) \quad cm^2/atom$$

$$\boxed{\sigma_{pair} \approx \frac{7}{9} \cdot \frac{A}{N_A} \cdot \frac{1}{X_0}}$$

Strong Interactions of Hadrons

Apart from electromagnetic interactions of charged particles, strong interactions can also be used in particle detection of hadrons. Strong interactions are important in detection of neutral hadrons.

One could have elastic scattering processes of the type,

$$\pi^- + p \rightarrow \pi^- + p$$

or inelastic reactions which produce new particles

$$\pi^- + p \rightarrow K^0 + \Lambda$$

At high energies, lots of inelastic reactions are possible, as long as conservation laws are obeyed.

Total cross section

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel}$$

where σ_{inel} is a sum over all possible inelastic processes that involve new particle production

The probability for a hadron-nucleus interaction as the hadron traverses a material of a small thickness dx, say, is,

$$n\,\sigma_{tot}\cdot dx$$

where, n is the number of nuclei per unit volume in the material.

The mean distance, or "Mean free path" travelled before an interaction occurs (or average distance between subsequent interactions) is

$$\lambda_c = \frac{1}{n\,\sigma_{tot}} \quad \leftarrow \text{collision length.}$$

Similarly, for inelastic processes,

$$\lambda_I = \frac{1}{n\,\sigma_{inel}} = \frac{A}{N_A\cdot\rho\cdot\sigma_{inel}}$$

(or λ_a for Absorption length)

$$n = \frac{N_A\cdot\rho}{A}$$

For N incident particles, number of interacti

$$dN = -N\cdot\frac{N_A\cdot\rho}{A}\cdot\sigma\cdot dx$$

$$\frac{dN}{N} = -\frac{dx}{\lambda_I} \qquad N(x) = N(0)\,e^{-x/\lambda_I}$$

Cross Section

← the probability for an interaction to occur

In the classical sense,

Cross-section ≡ geometrical cross sectional area of a sphere of a characteristic radius "a"

$$\sigma \sim \pi a^2$$

$$\sigma_{atom} \sim \pi a_0^2 \sim 3 \times 10^8 \text{ b} \qquad a_0 \sim 1 \text{ Å}$$

$$\sigma_{nucleus} \sim \pi a_N^2 \sim 30 \text{ mb} \qquad a_N \sim 1 \text{ fm}$$

$$1 \text{ b} = 10^{-24} \text{ cm}^2$$

$$\sigma_{atom} / \sigma_{nucleus} \sim 10^{10}$$

In quantum mechanics,

$$\sigma \sim |\langle f | H | i \rangle|^2$$

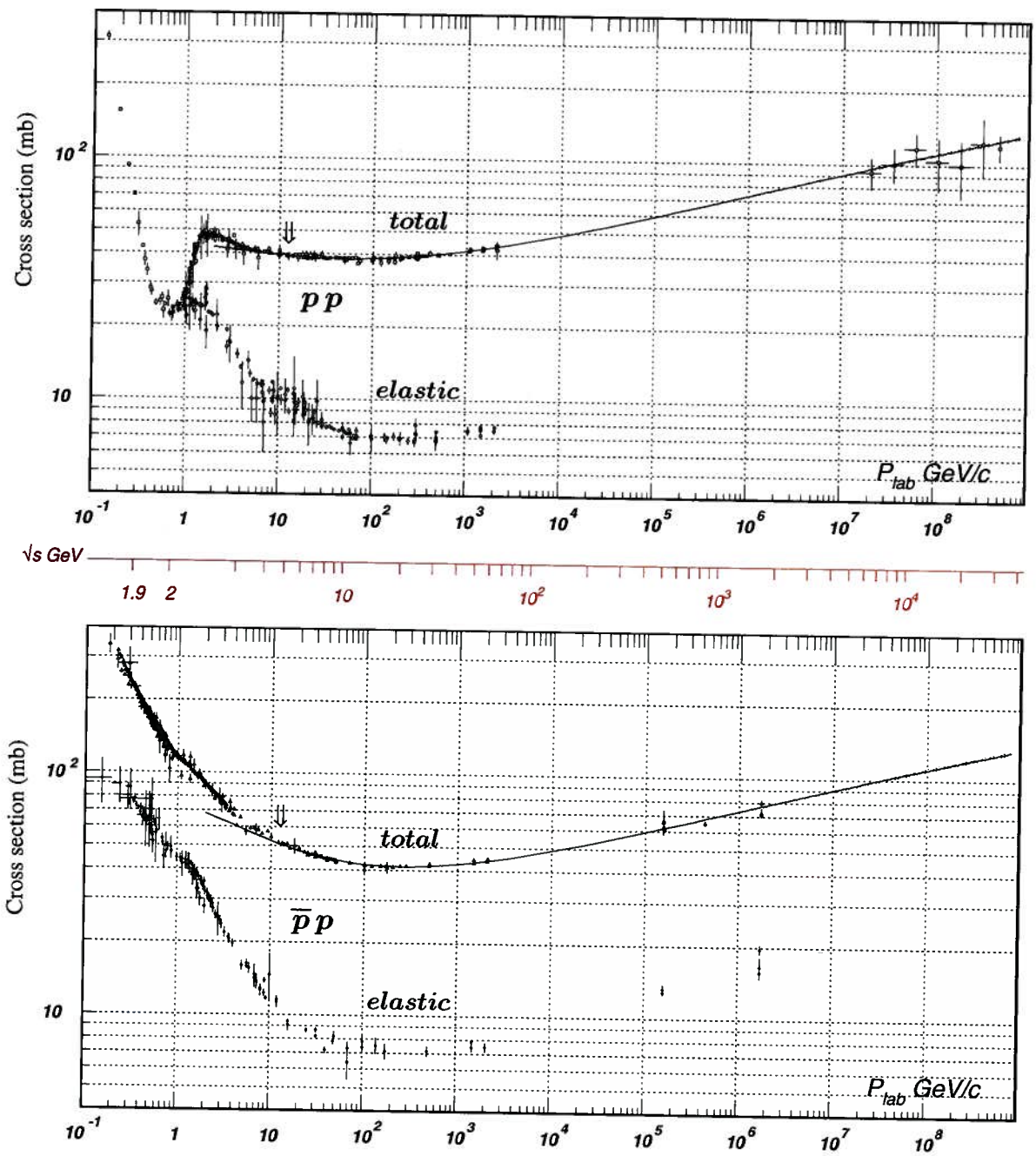


Figure 40.11: Total and elastic cross sections for pp and $\bar{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS group, IHEP, Protvino, August 2005)

Weak Interactions

Neutrinos and antineutrinos can also interact with nuclei, but very very rarely, as we have already talked about.

$$\bar{\nu}_\ell + p \rightarrow \ell^+ + X$$

The interactions rates are extremely small. For 1 MeV ν, for example, $\sigma \sim 10^{-47} m^2$. $\Rightarrow$ mean free path of millions of kms in matter. In water, $\lambda \sim 10^{20}$ cm.

When we study neutrino interactions, specifically we use huge amounts of material to observe multiple interactions, and look at interactions of produced particles in matter.

In other experiments, such as at the colliders, neutrinos are inferred from missing energy in a collision event.