Calculate the number of supports needed for the BTeV Forward Beam Pipe

Given: The design requirements as specified 17 September 2002:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube inner diameter</td>
<td>ID := 1.00  inch</td>
</tr>
<tr>
<td>Tolerance</td>
<td>IDtplus := 0.06 inch</td>
</tr>
<tr>
<td></td>
<td>IDtminus := 0 inch</td>
</tr>
<tr>
<td>Tube wall thickness</td>
<td>t := 0.013 inch</td>
</tr>
<tr>
<td>Tolerance</td>
<td>tplus := 0.002 inch</td>
</tr>
<tr>
<td></td>
<td>ttminus := 0.002 inch</td>
</tr>
<tr>
<td>Front z position</td>
<td>z1 := 29.0 inch</td>
</tr>
<tr>
<td>Back z position</td>
<td>z2 := 151.6 inch</td>
</tr>
<tr>
<td>Tube length</td>
<td>L := z2 − z1</td>
</tr>
<tr>
<td></td>
<td>L = 122.6 inch</td>
</tr>
<tr>
<td>Tube roundness</td>
<td>round := 0.020 inch</td>
</tr>
<tr>
<td>Tube straightness</td>
<td>st := 0.050 inch</td>
</tr>
<tr>
<td>Tube minimum clear line of sight</td>
<td>cls := 0.750 inch</td>
</tr>
<tr>
<td>Tube material: aluminum</td>
<td>E := 10 \times 10^6 psi</td>
</tr>
<tr>
<td></td>
<td>gamma := 170 lb/ft^3</td>
</tr>
</tbody>
</table>

Solution: First determine the maximum sag to see if it reaches a minimum clear line of sight.

Inner radius

\[ r_1 := \frac{ID}{2} \]

\[ r_1 = 0.5 \text{ inch} \]

Outer radius

\[ r_2 := r_1 + t \]

\[ r_2 = 0.513 \text{ inch} \]

Moment of inertia

\[ I := 0.25 \pi \left( r_2^4 - r_1^4 \right) \]

\[ I = 5.308 \times 10^{-3} \text{ inch}^4 \]

Distributed load per unit length

\[ w := \gamma \pi \left( \frac{r_2^2 - r_1^2}{12^2} \right) \]

\[ w = 0.049 \text{ lb/ft} \]

Maximum sag

\[ y := \frac{w L^4}{12 E I} \]

\[ y = 0.0054 \text{ inch} \]
Assuming that the inner diameter of the pipe remains perfectly round and straight and holds its inner diameter at 1-inch, the sag in the tube results in a minimum clear line of sight 0.812-inch.

However, if the tube is out-of-round, the minimum inner diameter of the tube is reduced to 0.99-inch. In addition, if the tube is not not straight by its maximum tolerance of 0.050-inch, the tube's inner diameter is further reduced to 0.89-inch. Taking into account the sag, the minimum clear line-of-sight becomes 0.702. Since the specified clear line-of-sight is 0.750, having a tube without a middle support is unacceptable.

Now, consider having the tube supported at z=125.0 inches so that the support lies between the fourth straw tube chamber and the fifth silicon strips detector. Calculate the minimum clear line-of-sight in the longest length of unsupported tube.

Longest length of unsupported tube
\[ L_1 := 125 - z_1 \]
\[ L_1 = 96 \quad \text{inch} \]

Maximum sag in unsupported tube
\[ y_1 := 0.0054 \frac{w}{12} \frac{L_1^4}{E \cdot I} \]
\[ y_1 = 0.035 \quad \text{inch} \]

Assuming a minimum tube inner diameter of 0.89-inch due to out-of-roundness and minimum straightness, adding in the sag in the unsupported tube results in a clear line-of-sight of 0.855-inch.

In conclusion, if the tube is not supported along its length, the combination of the tube out-of-roundness, minimum straightness, and maximum sag results in a clear line-of-sight of 0.702, which is less than the minimum allowed. However, if the tube supported at the position z=125 inches from C0, the clear line-of-sight is an acceptable 0.855-inch.