**Calculate the number of supports needed for the BTeV Forward Beam Pipe**

Given: The design requirements as specified 17 September 2002:

| | | |
|---|---|---|
| Tube inner diameter | $ID := 1.00$ | inch |
| Tolerance | $IDtplus := 0.06$ | inch |
| | $IDtminus := 0$ | inch |
| Tube wall thickness | $t := 0.013$ | inch |
| Tolerance | $ttplus := 0.002$ | inch |
| | ttminus:0.002 | inch |
| Front z position | $z1 := 29.0$ | inch |
| Back z position | $z2 := 151.6$ | inch |
| Tube length | $L := z2 - z1$ | |
| | $L = 122.6$ | inch |
| Tube roundness | $round := 0.020$ | inch |
| Tube straightness | $st := 0.050$ | inch |
| Tube minimum clear line of sight | $cls := 0.750$ | inch |
| Tube material: aluminum | | |
| Young's modulus | $E := 10 \cdot 10^{6}$ | psi |
| specific weight | $gamma := 170$ | lb/ft^3 |

Solution: First determine the maximum sag to see if it reaches a minimum clear line of sight.

| | | |
|---|---|---|
| Inner radius | $r1 := \frac{ID}{2}$ | |
| | $r1 = 0.5$ | inch |
| Outer radius | $r2 := r1 + t$ | |
| | $r2 = 0.513$ | inch |
| Moment of inertia | $I := 0.25 \cdot \pi \cdot \left(r2^{4} - r1^{4}\right)$ | |
| | $I = 5.308 \times 10^{-3}$ | inch^4 |
| Distributed load per unit length | $w := gamma \cdot \pi \cdot \frac{\left(r2^{2} - r1^{2}\right)}{12^{2}}$ | |
| | $w = 0.049$ | lb/ft |
| Maximum sag | $y := 0.0054 \cdot \frac{\frac{w}{12} \cdot L^{4}}{E \cdot I}$ | |
| | $y = 0.094$ | inch |

Assuming that the inner diameter of the pipe remains perfectly round and straight and holds its inner diameter at 1-inch, the sag in the tube results in a minimum clear line of sight 0.812-inch.

However, if the tube is out-of-round, the minimum inner diameter of the tube is reduced to 0.99-inch. In addition, if the tube is not not straight by its maximum tolerance of 0.050-inch, the tube's inner diameter is further reduced to 0.89-inch. Taking into account the sag, the minimum clear line-of-sight becomes 0.702. Since the specified clear line-of-sight is 0.750, having a tube without a middle support is unacceptable.

Now, consider having the tube supported at z=125.0 inches so that the support lies between the fourth straw tube chamber and the fifth silicon strips detector. Calculate the minimum clear line-of-sight in the longest length of unsupported tube.

Longest length of unsupported tube

$$L1 := 125 - z1$$

$$L1 = 96 \quad \text{inch}$$

Maximum sag in unsupported tube

$$y1 := 0.0054 \cdot \frac{\frac{w}{12} \cdot L1^4}{E \cdot I}$$

$$y1 = 0.035 \quad \text{inch}$$

Assuming a minimum tube inner diameter of 0.89-inch due to out-of-roundness and minimum straightness, adding in the sag in the unsupported tube results in a clear line-of-sight of 0.855-inch.

In conclusion, if the tube is not supported along its length, the combination of the tube out-of-roundess, minimum straightness, and maximum sag results in a clear line-of-sight of 0.702, which is less than the minimum allowed. However, **if the tube supported at the position z=125 inches from C0, the clear line-of-sight is an acceptable 0.855-inch.**

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