Effects of QCD bound states on relic abundance

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(based on...) work in progress with F. Luo (IPMU)

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Consider colored particle with mass

\[ m \gtrsim 1\text{TeV} \gg \Lambda_{\text{QCD}} \quad \text{in the early universe} \]

**Coulomb potential**

\[ V \sim \frac{\alpha_s}{r} \]

**Binding energy**

\[ E_B \sim \alpha_s^2 m \gtrsim 10\text{GeV} \]

**Inverse Bohr radius**

\[ a^{-1} \sim \alpha_s m \gtrsim 100\text{GeV} \]

we consider (perturbatively) QCD bound state way before QCD phase transition occurs, and its interaction with dark matter.
As an example, consider R-parity conserving Minimal Supersymmetric Standard Model (MSSM)

Consider the R-odd lightest SUSY particle (LSP) as the lightest neutralino $\chi_1$ and is the dark matter.
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Consider $\chi_1$ produced thermally.

$$\frac{dn_1}{dt} + 3Hn_1 = -\langle \sigma v \rangle_{11}(n_1^2 - n_1^{eq^2})$$
Standard DM relic abundance calculation

comoving DM number density

larger annihilation cross section -> smaller relic abundance

[Kolb, Turner ’90]
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Wino-like neutralino: $\sim 3 \text{ TeV}$

Higgsino-like neutralino: $\sim 1 \text{ TeV}$
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Bino?

depends on the masses of squarks & sleptons

usually bino is overproduced if sfermions are heavy
As an example, consider R-parity conserving Minimal Supersymmetric Standard Model (MSSM)

Consider the R-odd lightest SUSY particle (LSP) as the lightest neutralino $\chi_1$ and is the dark matter.

Consider $\chi_1$ produced thermally.

Specifically, consider LSP coannihilating with an almost mass-degenerate R-odd SUSY particle $\chi_2$ (not necessarily the second lightest neutralino). Coannihilation becomes vital.
How coannihilation works? [Griest, Seckel ’91]

conditions:

$\chi_2$ has large annihilation cross section with itself or $\chi_1$

\[
\chi_2\chi_2 \leftrightarrow SM SM \quad \chi_2\chi_1 \leftrightarrow SM SM
\]
How coannihilation works? [Griest, Seckel ’91]

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\[
\chi_2 \chi_2 \leftrightarrow SMSM \quad \chi_2 \chi_1 \leftrightarrow SMSM
\]

\( \chi_2 \) can convert to \( \chi_1 \) efficiently.

\[
\chi_2 SM \leftrightarrow \chi_1 SM
\]
Boltzmann equations

For simplicity, consider

\[
\frac{dn_1}{dt} + 2Hn_1 = -\langle \sigma v \rangle_{11}(n_1^2 - n_{1eq}^2)
\]

\[
\frac{dn_2}{dt} + 3Hn_2 = -\langle \sigma v \rangle_{22}(n_2^2 - n_{2eq}^2)
\]

fast conversion means that

\[
n_2/n_1 = n_{2eq}/n_{1eq} = \frac{g_2m_2^{3/2}}{g_1m_1^{3/2}} \exp\left(-\frac{(m_2 - m_1)}{T}\right)
\]

note that

\[
n_i^{eq} = g_i \left(m_i T/2\pi\right)^{3/2} e^{-m_i/T}
\]
Boltzmann equations

assuming fast conversion    \[ \chi_2 S M \leftrightarrow \chi_1 S M \]

defining       \[ n \equiv n_1 + n_2 \]

\[
\frac{dn}{dt} + 3Hn = - \sum_{i,j=1}^{2} \langle \sigma v \rangle_{i,j \rightarrow SM} \frac{n_{i \text{eq}} n_{j \text{eq}}}{n_{\text{eq}}^2} (n^2 - n_{\text{eq}}^2)
\]

call this \[ \langle \sigma v \rangle_{\text{eff}} \]

compare with

\[
\frac{dn_{\chi}}{dt} + 3Hn_{\chi} = - \langle \sigma v \rangle_{\chi \chi \rightarrow SM} \left( n_{\chi}^2 - n_{\chi \text{eq}}^2 \right)
\]

without coannihilation
\[
\frac{dn}{dt} + 3Hn = -\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij \rightarrow SM} \frac{n_{i}^{eq} n_{j}^{eq}}{n_{eq}^{2}} (n^2 - n_{eq}^2)
\]

Two limits

\[m_2 \gg m_1: \langle \sigma v \rangle_{\text{eff}} \approx \langle \sigma v \rangle_{11 \rightarrow SM}\]

\[m_2 = m_1: \langle \sigma v \rangle_{\text{eff}} = \frac{g_1^2 \langle \sigma v \rangle_{11 \rightarrow SM} + g_2^2 \langle \sigma v \rangle_{22 \rightarrow SM} + 2g_1 g_2 \langle \sigma v \rangle_{12 \rightarrow SM}}{(g_1 + g_2)^2}\]

note that \[n_{i}^{eq} = g_i (m_i T / 2\pi)^{3/2} e^{-m_i / T}\]
We consider dark matter accompanied by an almost mass-degenerate colored particle.
If $\chi_2$ is colored (squark or gluino in MSSM) QCD Sommerfeld effect is important

\[
\begin{align*}
\chi_2 & \rightarrow g \\
\chi_2 & \rightarrow g
\end{align*}
\]

tree-level annihilation

\[
\begin{align*}
\chi_2 & \rightarrow g \\
\chi_2 & \rightarrow g
\end{align*}
\]

non-perturbative (Sommerfeld) effect that modifies the initial-state wave function

see e.g. [De Simone et al. '14]
If $\chi_2$ is colored (squark or gluino in MSSM) formation of QCD bound state of $\chi_2$ could be important as well.

\[ \tilde{g}\tilde{g} \leftrightarrow \tilde{R}g, \tilde{R} \leftrightarrow gg \quad \text{for gluino} \]

\[ \tilde{t}\tilde{t} \leftrightarrow \tilde{\eta}g, \tilde{\eta} \leftrightarrow gg \quad \text{for stop} \]

Compare recombination process $e^- p \leftrightarrow H\gamma$ [Ellis et al. '15]
If $\chi_2$ is colored (squark or gluino in MSSM) formation of QCD bound state of $\chi_2$ could be important as well.

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Compare recombination process $e^- p \leftrightarrow H\gamma$

note: bound state formation is important only when

\[ \Gamma_{\text{ann}} \gtrsim \Gamma_{\tilde{t}/\tilde{g}} \]

bound state annihilation rate decay rate
note that bound state annihilation removes 2 R-odd particles, thus helps reducing DM density

 gluino bound state  \( \tilde{R} \leftrightarrow gg \)
 stop bound state  \( \tilde{\eta} \leftrightarrow gg \)
call the colored particle $X$ and bound state $\eta$

one needs to solve the coupled Boltzmann eq. including the bound state $\eta$

$$\frac{dn_\eta}{dt} + 3H n_\eta = -\Gamma_\eta(n_\eta - n_{\eta}^{eq}) + \Gamma_{bsf}(n_X^2 - n_X^{eq2} \frac{n_\eta}{n_{\eta}^{eq}})$$

bound state annihilation rate

bound state formation rate

bound state dissociation rate
Solving the coupled Boltzmann equations

\[ \frac{dn_1}{dt} + 2H n_1 = -\langle \sigma v \rangle_{11} (n_1^2 - n_{1eq}^2) \]

\[ \frac{dn_X}{dt} + 3H n_X = -\langle \sigma v \rangle_{XX} (n_X^2 - n_{Xeq}^2) - \Gamma_{bsf} (n_X^2 - n_{Xeq}^2 \frac{n_\eta}{n_{\etaeq}}) \]

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bound state number density is exponentially suppressed. One can set LHS to zero as an approximation. (the validity of this approx. has been checked numerically)
Solving the coupled Boltzmann equations

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\]
Then, the Boltzmann equation is modified by adding the following terms:

\[
\frac{dn}{dt} + 3Hn \simeq - \sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij \rightarrow SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2} (n^2 - n_{eq}^2)
\]

\[
- \langle \sigma v \rangle_{XX \rightarrow \eta g} \frac{\langle \Gamma \rangle_{\eta \rightarrow gg}}{\langle \Gamma \rangle_{\eta \rightarrow gg} + \langle \Gamma \rangle_{\eta g \rightarrow XX}} (n_{X}^2 - n_{X}^{eq2})
\]

- **bound state formation rate**
- **bound state annihilation rate**
- **bound state dissociation rate**
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\]

\[
- \left( \langle \sigma v \rangle_{XX \rightarrow \eta g} \frac{\langle \Gamma \rangle_{\eta \rightarrow gg}}{\langle \Gamma \rangle_{\eta \rightarrow gg} + \langle \Gamma \rangle_{\eta g \rightarrow XX}} \right) (n_X^2 - n_{eq}^2X)
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\eta g \rightarrow XX \text{ becomes unimportant at low temperature compared to } \eta \rightarrow gg

because gluon is not energetic enough to dissociate the bound state
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(at temperature T < binding energy)

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\frac{d n}{d t} + 3 H n \simeq - \sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij \to SM} \frac{n_{i}^{eq} n_{j}^{eq}}{n_{eq}^2} (n^2 - n_{eq}^2)
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- \langle \sigma v \rangle_{XX \to \eta g} \frac{\langle \Gamma \rangle_{\eta \to gg}}{\langle \Gamma \rangle_{\eta \to gg} + \langle \Gamma \rangle_{\eta \to XX}} (n_{X}^2 - n_{X}^{eq^2})
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late-time “annihilation” is important! One needs to solve
the Boltzmann eqs. numerically
Calculation of bound state formation/dissociation rate
Calculation of bound state formation/dissociation rate

Use Coulomb approximation to describe the bound state

\[ V(r) = -C \frac{\alpha_s}{r} \]

with \[ C = \frac{1}{2} (C_1 + C_2 - C_{(12)}) \]

SU(3) quadratic casimir of constituent particle \quad SU(3) quadratic casimir of bound state
Use Coulomb approximation

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with

\[ C = \frac{1}{2} (C_1 + C_2 - C_{(12)}) \]

<table>
<thead>
<tr>
<th>MSSM</th>
<th>binding</th>
<th>non-binding</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{g}\tilde{g}$</td>
<td>$1, 8_S, 8_A$</td>
<td>$10, 10, 27$</td>
</tr>
<tr>
<td>$\tilde{t}\tilde{t}^*$</td>
<td>$1$</td>
<td>$8$</td>
</tr>
<tr>
<td>$\tilde{t}\tilde{t}$</td>
<td>$3$</td>
<td>$6$</td>
</tr>
<tr>
<td>$\tilde{t}\tilde{g}$</td>
<td>$3, 6$</td>
<td>$15$</td>
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<td>$3/2$</td>
</tr>
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<td>$(\tilde{t}\tilde{t}), (\tilde{t}^<em>\tilde{t}^</em>)$</td>
<td>$\bar{3}, 3$</td>
<td>$2/3$</td>
</tr>
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<tr>
<td></td>
<td>$\bar{6}, 6$</td>
<td>$1/2$</td>
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Consider photoelectric effect as an analogy

photoelectric effect:  \[ H\gamma \rightarrow e^- p \]
Consider photoelectric effect as an analogy

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photoelectric effect:

Electromagnetic Hamiltonian

\[ H = \frac{1}{2m} (\vec{p} + e \vec{A})^2 \]
Consider photoelectric effect as an analogy

photoelectric effect: \[ H \gamma \rightarrow e^- p \]

Electromagnetic Hamiltonian:

\[ H = \frac{1}{2m} (\vec{p} + e\vec{A})^2 \]

\[ H \approx \frac{p^2}{2m} + \frac{e}{m} \vec{A} \cdot \vec{p} \]
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calculate the matrix

\[
\langle \phi_f | \frac{e}{m} \vec{A} \cdot \vec{p} | \phi_i \rangle
\]

free particle wave function

bound state wave function
Consider photoelectric effect as an analogy

photoelectric effect: \[ H \gamma \rightarrow e^- p \]

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\]

calculate the matrix \[ \langle \phi_f | \frac{e}{m} \vec{A} \cdot \vec{p} | \phi_i \rangle \]

rescale with appropriate color factors
Bound state formation rate is related to the dissociation rate via the **Milne relation** (or principle of detailed balance).

\[ n_{X_1}^{eq} n_{X_2}^{eq} \sigma_{bsf} v_{rel} \left( 1 + \frac{1}{e^{\omega/T} - 1} \right) f(v_{rel}) dv_{rel} = n_{\eta}^{eq} \sigma_{dis} d\eta_{g}^{eq} \]
scalar triplet bound state (Stoponium)

\[ \bar{t}t \rightarrow g\eta_{\bar{t}} \]

\[ E_B = \left( \frac{4}{3} \alpha_s \right)^2 \left( \frac{m_{\bar{t}}}{2} \right) / 2, \]

\[ a^{-1} = \left( \frac{4}{3} \alpha_s \right) \left( \frac{m_{\bar{t}}}{2} \right), \]

\[ \nu = \left( \frac{1}{6} \alpha_s \right) / \nu_{rel}, \]

\[ \sigma_{\text{dis}}^0 = \frac{2^6 \pi^2}{3} \alpha_s a^2 \left( \frac{E_B}{\omega} \right)^4 \frac{1 + \nu^2}{1 + (8\nu)^2} \frac{e^{4\nu \cot^{-1}(8\nu) - 2\pi\nu}}{1 - e^{-2\pi\nu}}, \]

\[ \sigma_{\text{dis}} = \frac{4}{3} \times \frac{1}{8} \times \sigma_{\text{dis}}^0 \]

\[ \sigma_{\text{rec}} = \frac{4}{9} \left( \frac{4}{3} \alpha_s \right)^2 (1 + (8\nu)^2)^2 (8\nu)^{-2} \sigma_{\text{dis}} \]

we consider only the ground state

dissociation rate

formation rate
Scalar triplet (stop) coannihilation

Colored bands show parameter region matching the observed DM relic abundance

DM-stop mass splitting

DM mass
Scalar triplet (stop) coannihilation

Colored bands show parameter region matching the observed DM relic abundance

- no Sommerfeld effect
- no bound-state effect
- Sommerfeld effect
- no bound-state effect
- Sommerfeld plus bound-state effect
- Sommerfeld plus 2x bound-state effect
Coannihilation with other types of colored particle

(fast conversion implicitly assumed)

fermion triplet

fermion octet (gluino)

scalar octet
gluino coannihilation
(with conversion taken into account appropriately)

\[ m_q/m_g = 1.1 \]

\[ m_q/m_g = 10 \]

\[ m_q/m_g = 50 \]

\[ m_q/m_g = 120 \]

[Ellis et al. ‘15]
a short comment on 100 TeV collider prospects

bino/stop coan. 5-sigma discovery becomes impossible at 100 TeV collider

[Low, Wang ′14]
bino/stop coan. 5-sigma discovery becomes impossible even at 100 TeV collider

significance

[Low, Wang ‘14]
Other implications of bound-state effects: BBN constraints on long-lived particles

\[ \frac{n_{\tilde{t}}}{s} \]

\[ m_{\tilde{t}} \ [\text{GeV}] \]

\[ \tau_{\tilde{t}} \sim 0.1 - 10^2 \text{s} \]

\[ \tau_{\tilde{t}} \sim 10^2 - 10^7 \text{s} \]

see e.g. [Kawasaki et al ‘04]
Summary

We have considered dark matter accompanied by an almost mass-degenerate colored particle.

Bound state of the colored particles can increase the effective annihilation cross section significantly.
Backup
How large are bound-state effects?

For gluino

$$\frac{\sigma_{bsf} v_{rel}}{S_{ann}(\sigma_{ann} v_{rel})} \sim 1.4 \quad (v_{rel} \to 0)$$

For stop

$$\langle \frac{\sigma_{bsf} v_{rel}}{\sigma_{ann} v_{rel}} \rangle$$

- Attractive-repulsive

![Graph showing attractive-repulsive behavior with curves for different values of $\kappa$.](image)

$E_B/T$: binding energy/temperature